Dwelling Substitutability and the Delineation of Submarkets

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ABSTRACT

This paper aims to stimulate a step-change in how and why submarkets are analysed. Recent work on submarkets has focussed on their potential for improving prediction accuracy but submarkets may also provide valuable insights into urban housing market structures and how they interact with social and spatial processes at the local level. The paper attempts to establish a set of criteria that submarket methodologies should meet in order to investigate the nature and meaning of submarkets in a more robust and purposeful way. Existing approaches are critically evaluated using these criteria, and an alternative methodology is proposed, grounded in the notion of submarkets as a function of substitutability, with a view to helping researchers address a richer set of questions regarding housing submarkets. The approach is illustrated using data on Glasgow.

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1. Introduction

What causes two dwellings to be close substitutes? Similarity of attributes, construction type, style and quality of finish are all obvious drivers. It has become common to define submarkets accordingly – either as collections of dwellings with similar attributes or with similar attribute prices.[1] Such definitions are problematic, however, because very different dwellings, located in different locations, can be substitutes. A particularly important paradox emerges when unequal access to particular amenities leads to two dwellings having different attribute prices, even though they are considered by the market to be close substitutes. The *amenity interaction paradox* (AIP) arises because very different combinations of attributes and amenities can provide similar levels of utility. A family with young children may be indifferent between a large suburban house with a garden, and a small central flat located near a public park with recreational facilities. Someone with a passion for the great outdoors may choose to live on the edge of the city, or they may equally locate near a transport hub allowing them to escape to rural destinations. Such interactions make it impossible to know whether observed differences in marginal prices are due to genuine submarket fissures or the result of omitted interaction effects, or simply irrelevant to the question of substitutability.

The determination of substitutability is complicated further by the effect of social and ethnic mix. Even weak preference for neighbourhood homogeneity (such as aversion to being in the minority) can unleash powerful forces of segmentation (Schelling 1971). The housing market is not detached from this process. Whether two dwellings are considered close substitutes will be influenced by the social composition of their respective neighbourhoods and by the preferences of market participants. How these different forces interact to determine substitutability is complex, subject to the whim, perception and prejudice of consumers and the myriad ways their heterogeneous needs and preferences can be met by an equally multi-dimensional set of property types, location and social mix.

The concept of submarkets emerged in the largely qualitative writings of Rapkin *et al.* (1953) and Grigsby (1963)**,** and given a quantitative interpretation by Rothenberg *et al.* (1991), grounded in the concept of substitutability. A substitutability approach to submarkets says that properties are in the same submarket if they are considered *by the market* to be close substitutes. Note the emphasis. This is not the same as saying two dwellings are in the

[1] See review by Watkins (2001).

same submarket if they have similar physical attributes or similar access to quality schooling. Such features may or may not play a significant role in how the market groups dwellings. There may be as many combinations of dwelling type and location as there are dwellings; and the *interactions* of different combinations may be at least as important as the features themselves in how the market views them.

An essential feature, therefore, of any method of identifying submarkets is that it does not make assumptions about how the market decides on substitutability. We should seek a method that does not impose submarket structures (by clustering dwellings by location or physical form, for example), but rather reveals them. Such a method could unearth findings of great interest. For example, it would be intriguing to see the extent to which distant dwellings are considered close substitutes, and to observe the geographical shape and pattern of submarkets. Shape is potentially important as it could raise some deep theoretical questions. If, for example, submarkets have long, serpentine, boundaries, does this imply an abundance of households content to live at the frontier between contrasting communities? Conversely, if submarkets are compact and spherical, would this imply that indifferent households are scarce? (i.e. most prefer homogeneity and so market sorting minimises boundary length).

Then there are related questions such as the extent to which submarkets are interconnected, gradually merging into each other across space, and the extent to which they bounded by discrete frontiers. Are submarket boundaries, in fact, an arbitrary concept? Should we rather think in terms of a continuous surface connecting all dwellings in a “lattice of substitution” where peaks represent close substitutes? This is the view presented here, where submarkets are a derived construct, a stylised simplification the complex, multi-layered landscape of substitution.

Crucially, such questions demand a method of measuring substitutability, the absence of which perhaps explains why the literature has drifted from the original themes of Rapkin and Grigsby towards viewing submarkets as a means of improving hedonic prediction performance (Bourassa *et al.* 2003), and why most recent studies rely on homogenous attribute price vectors (HAPV), rather than substitutability, as the means of identifying submarket boundaries (see the review by Watkins 2001), or as the assumption underpinning a hedonic index of housing quality (Rothenberg *et al.* 1991).

This paper attempts to step back from this empirical consensus and consider the qualities we should require of a submarket methodology. The discussion is woven around a series of proposed criteria and supporting rationale, which lead to the conclusion that existing approaches to submarket delineation have a weak theoretical base. An alternative method is offered in section three, grounded directly in the notion of substitutability. Rather than testing for (or relying on) HAPVs, it is proposed that dwellings should be grouped on the basis of substitutability, measured using the Cross Price Elasticity of Price (CPEP) which is shown to have a one-to-one mapping with the Cross Price Elasticity of Demand (CPED). Section four considers how CPEP could be used to explore the existence and spatiality of submarkets. Section five illustrates the method using real data (house sales in Glasgow, Scotland). Section five concludes.

2. What do we require from Submarket Estimation Methods?

This section sets out the notation required for the subsequent analysis and then summarises the main strands of the argument before listing and justifying a series of qualities we seek in a submarket estimation method. Consider the following inventory of housing market entities and definitions:

Household:	$b = 1, 2, \ldots B$
Dwellings (or blocks of dwellings):	$i = 1, 2, \ldots V$
Attribute vector for dwelling i:	$\mathbf{z}_i = z_{(1)i}, z_{(2)i}, \ldots z_{(A)i}$
Attribute price vector for dwelling i:	$\mathbf{P}(\mathbf{z}_i) = P_{(1)i}, P_{(2)\,i}, \ldots P_{(n)i}$
HAPV:	$\mathbf{P}(\mathbf{z}_i) = \mathbf{P}(\mathbf{z}_j)$
Submarkets:	$\mathrm{S}_k \subseteq \mathbf{M}$ where $k = 1, 2, \ldots K$

M is the family of submarkets that make up the urban housing market as a whole. Each dwelling (or block of dwellings) is an element of a submarket and of the wider housing market:

$$i \in \mathrm{S}_k \;\Rightarrow\; i \in \mathbf{M}$$

If there are no submarkets, only a single uniform housing market, then $K = 1$ and,

$$\mathrm{S}_1 = \mathbf{M} \qquad [1]$$

For $K > 1$, S_k is defined by some criterion that allocates dwellings (or blocks of dwellings) to different subsets of **M**. It is assumed that this criterion is either synonymous with, or analytically equivalent to, the notion of substitutability between dwellings (or whatever spatial units are being considered), and leads to the partitioning of **M** so that it can be described as a composite of separate, but inter-connected, submarkets. **M** equals the union of all submarkets,

$$\mathbf{M} = \bigcup_{1}^{K} S_k = S_1 \cup S_2 \cup ... \cup S_K \qquad [2]$$

Because **M** is a partitioned set, all submarkets are disjoint, and a house cannot be a member of more than one submarket:

$$S_k \cap S_l = \varnothing \quad \forall\ k \neq l \qquad [3]$$

We can now summarise the conventional logic of submarket testing as follows:

If HAPV then $i,j \in S_k$

HAPV

―――――

$\therefore\ i,j \in S_k$ (*modus ponens*)

There is a problem with the first premise. Similarity of attribute prices does *not,* in fact, imply that two dwellings (or two blocks of dwellings) are in the same submarket. *i* could be located in Amsterdam, and *j* in Glasgow. That they have similar attribute prices at a given point in time is coincidental – the two properties are highly unlikely to fall into the same choice set of any one buyer, *b*, and cannot meaningfully be described as being close substitutes or as belonging to the same submarket.

A second strand of conventional logic states that if two dwellings are elements of the same submarket, then they will have the same attribute prices:

If $i,j \in S_k$ then HAPV

$i,j \in S_k$

―――――

$\therefore$ HAPV

Application of *modus tollens* allows us to deduce that if *i* and *j* do not have the same attribute prices then they will not be elements of the same submarket:

If $i,j \in S_k$ then HAPV

¬HAPV

―――――

$\therefore\ i,j \notin S_k$ (*modus tollens*) where "¬" represents negation. In other words, HAPV provides a necessary but not a sufficient condition for two dwellings to be in the same submarket. Unfortunately, this second strand of logic also falters because of Transformative Interaction Effects (TIEs) and Many to Many Mappings of Means and Ends (MMMEs), explained below. We should therefore also doubt the potency of HAPV as a necessary condition. These and other problems (such as the effect of attribute measurement errors) lead us to further question the legitimacy of relying on HAPV as a sufficient condition: if one observes HAPV, how does one know that all relevant attributes have been included, or whether one has truly captured transformative interaction effects? HAPV can be a false indicator of common membership of a submarket in the same way that ¬HAPV can be a false indicator of market segmentation.

These arguments are now considered in more detail, woven around a set of proposed criteria for evaluating submarket estimation methods (SEMs). In summary, it is argued that SEMs should be robust to *1. the contingent nature of attribute effects; 2. the continuity of substitutability space; 3. unobserved attribute variation; and 4. non-convexity, non-compactness, and non-contiguity*

Criterion 1: SEMs should be Robust to the Contingent Nature of Attribute Effects

<u>Rationale</u>: *The existence of Many to Many Mapping of Means and Ends* (*MMMEs) and Transformative Interaction Effects (TIES) implies that properties can be in the same submarket but have different attribute prices. I.e.* MMMEs $\Rightarrow \exists\ i,j$ *such that* $[i,j \in S_k] \wedge [\mathbf{P}(\mathbf{z}_i) \neq \mathbf{P}(\mathbf{z}_j)]$.

MMMEs exist because the same human need can be met in different ways ("there is more than one way to skin a cat"), and the same means will meet different needs for different people ("one man's meat is another man's poison" – i.e. consumer preferences are heterogeneous). Consequently, two goods can have very few common attributes, and very divergent attribute prices, yet still be considered close substitutes.

An important driver of MMMEs, and one that makes predicting the global set of MMMEs considerably more elusive, is the existence of *Transformative Interaction Effects* (TIEs). TIEs occur when the effect of an attribute (whether geographical or structural) is fundamentally transformed when placed in a particular context, such as being combined with another attribute (either geographical or structural).

To illustrate, consider the options for crossing the English Channel.[2] Assume that ferries, planes and trains are all close substitutes – a change in the price or availability of one will have a large effect on the demand for the others (they are all in the same market for transferring passengers across the English Channel).[3] While each mode of transport has contrasting physical features, the features combine to offer a similar *service* because of TIEs. This leads us to question the meaningfulness of relying on attribute price differences as a gauge of substitutability or, indeed, submarkets. For example, whereas wings on trains are of no value, wings on aeroplanes are rather essential. Presumably, the attribute price of wings would differ dramatically between trains and planes because the utility of wings is transformed when appropriately combined with a jet engine and an aerodynamic fuselage. Similarly, wheels on ferries will be valueless, yet vital to the functioning of trains. Note that, if we were to persist with the application of HAPV as our means of submarket delineation, the divergent attribute prices on wings would lead us to the erroneous conclusion that the different modes of transport are very distant substitutes, belonging to separate submarkets. The error arises because, although the physical attributes of an aeroplane are quite different to those of a ferry, the *final service bundle* (comfortable and speedy travel) offered by the particular combination of attributes that each mode of transport entails is very similar. Decomposing transport modes into their constituent physical parts is unhelpful when considering their substitutability – what counts is the final service bundle they offer, which is remarkably similar, despite the structural heterogeneity.

A special case of TIEs, and one that is of particular relevance to housing, is the *amenity interaction paradox,* mentioned in the introduction. AIP arises from the transforming effect of geographical context on the value and role of particular attributes, an effect that causes dissimilar combinations of characteristics to be close substitutes and/or similar combinations to be distant substitutes. Location brings a long list of potential sources of utility – proximity to open space and environmental features, to sources of employment, to retail outlets, to crime, to social networks, to schooling, to religious and cultural centres. These all affect and interact with a heterogeneous set of preferences among consumers. The effects of territoriality (Kintrea *et al.* 2008), stigma and area esteem are also potentially important in shaping location choice, as are preferences for racial and social mix (Schelling 1971). And preferences for *dwelling* attributes are also likely heterogeneous. Demand is shaped not only by the shelter that housing offers but also by fashion, lifestyle aspiration, and the social

[2] Or *French* Channel, depending on one's perspective…

[3] Anguera (2006) records that as the number of Channel Tunnel passengers increased from 0.1 million in 1994 to 6.3 million, the number of ferry passengers fell from 23.7 million to 16.6 million over the same period.

connotations of dwelling design – a house is “seen as an expression of our taste and as an extension of our personality” (Sweet, 1999, p. 15).

All this leads to a final service bundle with many more dimensions than that of our transport example, and to a heady set of possible combinations of dwelling and location attributes considered close or distant substitutes by individual consumers. Two dwellings in Belfast might be identical and exist in close proximity but be considered distant substitutes because one is located in a Protestant street, the other Catholic. Picture windows can be of great value when a house is not overlooked and/or has spectacular views, but rather less desirable in a dwelling with little privacy and/or an unsightly outlook. Proximity to public swimming facilities is more desirable if one does not have a pool of one’s own. Yet, dwellings fitted with pools may still be considered close substitutes by those who want to swim, even though differences in proximity to public pools may lead to attribute price variation. Ostensibly, a house with a large garden may appear to meet a very different need to a house with a small garden. However, for a family with young children, the two may be actually be considered close substitutes if the latter is located near a public park. A home with excellent access to an employment node may be considered a close substitute for a dwelling without particularly good access to any one employment node if that house has adequate access to many nodes. This is because of “the expectation of where future jobs will be and the likelihood of both job separations and residential moves” (Crane, 1996, p.342).

The point is that MMMEs and TIEs may be subtle and unforeseen because the service bundle provided by a house and its location is so multi-faceted, and because different combinations of physical and locations attributes can interact in different ways to meet different needs for different people. If we use a HAPV approach, we shall never know whether observed breaks in attribute prices reflect genuine differences in service bundle or are simply irrelevant to the question of substitutability, in the same way that the price (or indeed existence) of wings on automobiles vs. price (existence) of wings on aeroplanes is irrelevant to the substitutability of transport modes.

<u>Corollary 1:</u> Cluster methods are theoretically problematic if applied to dwelling attributes (either structural or geographical) rather than behavioural variables.

<u>Rationale</u>: Properties can be in the same attribute cluster group but not be in the same submarket, and they can be in the same submarket but not be in the same attribute cluster group. I.e. MMMEs $\Rightarrow$ *(a)* $\exists\ i, j, g_h$ *such that* $[i,j \in g_h] \wedge [i,j \notin S_k]$ *; and (b)* $\exists\ i, j$ *such that* $[i,j \notin g_h] \wedge [i,j \in S_k]$; *where* g_h *is a group of properties defined by a cluster function applied to attribute vector* $\mathbf{z}$.

Statistical clustering of dwellings by physical attributes has become popular in the submarkets literature,[4] but may not reflect how consumers group them, which may be nuanced and difficult to anticipate, not least because of the interplay between MMMEs, TIES and the effects of imperfect information (such as the conventions that emerge in search patterns and channels of communication – see Pryce and Oates, 2008, p.325, 337-341) which may be crucial in determining which dwellings enter the choice set of a given consumer.

We would like to group properties according to a variable that captures market behaviour (e.g. substitutability), rather than the properties of the dwelling stock. In transactions data, the only behavioural variable actually measured is the selling price, but this is the dependent variable and is typically excluded from the clustering process. Similarly, grouping variables using factor analysis imposes a structure on the functional form of the hedonic equation in a way that removes the possibility of capturing how the market views the interplay of attributes in determining prices.[5]

We should note that grouping properties by type of resident is also problematic: while two consumers may have different preference maps, this does not mean they will disagree on whether two properties are close substitutes – they may consider the dwellings to be substitutable but for very different reasons. And even if we were able to anticipate individual

[4] Researchers typically apply principle component, cluster or factor analysis to bunch properties into product groups on the basis of physical characteristics. Dwellings within a particular group are viewed as substitutes. Hedonic price regressions are then run on each product group separately leading to improved regression fit and prediction accuracy. Maclennan and Tu (1996), for example, use principle components analysis to identify the key variables that explain variation in their data on Glasgow, and then apply cluster analysis to those variables. Bourassa *et al* (1999) follow a similar process using principle component analysis to extract a set of factors from the original set of variables from local government area and individual dwelling data on Sydney. They then apply cluster analysis to the scores of the most important factors to determine the segmentation of submarkets and finally run hedonic price regressions on the subsamples to show that the clustering procedure results in a model that is "significantly better than classifications derived from all other methods of constructing housing submarkets" (p.160). Further examples include Goodman and Thibodeau (1998) and Leishman (2009).

[5] This is an example of a general problem associated with cluster analysis and principle components methods – Greene (1993), for example, questions the usefulness of principle components because "the principle components are not chosen on the basis of any relationship of the regressors to *y*, the variable we are attempting to explain" (p.273).

rankings of substitutability, the outcome at the level of the market is made fundamentally unpredictable by the *Condorcet paradox*[6] – transitivities that hold at the level of the individual do not necessarily hold in the aggregate.

One would like to cluster by market-level substitutability, but this requires a way of measuring it, and this has hitherto eluded submarket researchers.

Corollary 2: *Spatially correlated errors provide neither a necessary nor a sufficient condition for submarkets*.

Rationale: *MMMEs* $\Rightarrow$ (*a*) $\exists$ *i*, *j such that* $[C(e_i, e_j) > c^*] \wedge [i,j \notin S_k]$; *and* (*b*) $\exists$ *i,j such that* $[i,j \in S_k] \wedge [C(e_i, e_j) < c^*]$, *where e is the hedonic price equation residual for the property, C is some measure of spatial correlation, and c* is an accepted threshold above which errors are deemed to be spatially correlated.*

Because goods can be close substitutes but have different attributes, there is no reason to believe that spatial patterns in "uncaptured non-linear relationships between the dependent and independent variables" (Tu *et al.* 2007 p. 388) in a hedonic regression will have any bearing on where fissures in substitutability lie. Even if one were able to measure, without error, all the physical and amenity differences between dwellings, it remains possible that very different bundles of physical and location attributes are actually perceived to be close substitutes by consumers – it is the utility of the inseparable and idiosyncratic fusion of attributes that a buyer is purchasing, not the linear sum of components.

Spatially auto-correlated errors as a measure of substitutability may be further distorted by the arbitrary nature of uncaptured nonlinearities. For example, for block *i*, our estimation may do well at capturing them; not so for block *j*. So, spatial clusters of errors may not reveal HAPV boundaries but patterns of uncaptured nonlinearities. Further difficulties arise from the effect of non-convexity, granularity and non-compactness in the shape of submarkets, and the spatial clustering of attribute measurement errors (discussed below).

[6] See, for example, Arrow's (1950) application to the majority voting problem.

Criterion 2: SEMs should be Robust to the Continuity of Substitutability Space

Rationale*: Continuous differentiability of the substitution function linking i and j in Cartesian space does not preclude i and j from belonging to separate submarkets. I.e. it is possible that* $\exists$ *i,j and* $\eta_{ij} = f(x,y)$ *such that* $[f^{(n)}(x,y)$ *exists*$] \wedge [i,j \notin S_k]$*, where x and y are Cartesian coordinates and* $n > 0$ *is the number of times that* $f(x,y)$ *can be differentiated.*

The amenity interaction paradox arises, in part, because of a weak definition of submarkets – one that relies on shifts in attribute prices. This definition is problematic because it requires one to have a theory of why attribute prices remain heterogeneous. Goodman and Thibodeau (1998) argue that attribute price differences persist because of amenity effects. However, this suggests that heterogeneous attribute prices (and hence submarkets) only exist because of omitted variables. In principle, therefore, if one were to construct a model that captures amenity affects (both social and economic), there would be no difference in attribute prices, and no such thing as submarkets!

But, if the utility of a house is a function of its location and structural attributes, why disentangle location and structural attributes for the purposes of submarket definition? Using Rosen's (1974) terminology, we are considering a class of commodities – i.e. houses – that are described by *n* attributes or characteristics, $\mathbf{z} = (z_{(1)}, z_{(2)}, \ldots, z_{(A)})$. The conventional definition of submarkets is weak because the entire notion of submarkets can be subsumed by simply allowing **z** to include a mixture of structural *and* location attributes, along with interactions between the two.

An alternative justification of persistent attribute price differences is to assume that they are caused by market inefficiencies and frictions (such as imperfect competition among households and the inelastic demand and supply of housing service, *cf*. Schnare and Struyck 1976). While this is feasible, it leads again to a weak definition because it means that submarkets only exist if markets are inefficient or inflexible. This is problematic because variation in substitutability of dwellings could persist, even in a world of perfectly efficient markets. Heterogeneous dwelling types, heterogeneous locations, heterogeneous preferences, many to many mappings of means and ends, and transformative interaction effects of structural and location features, can all exist in a frictionless world, and would cause substitutability to vary across dwellings, even if the supply of each dwelling type was perfectly elastic and all market participates were perfectly informed. Heterogeneity, not market inefficiency, should be the primary basis for our theory of substitutability (and hence of submarkets), not market imperfections.

This means that a test for submarkets should not be assumed to synonymous with a test for inefficiency (as in Schnare and Struyck 1976) and neither should it be seen as synonymous with a test for discrete breaks in the land rent surface (see, for example, Fik *et al.* 2003 p.635, 638). If we assume the latter, then we are vulnerable to the corollary that a world without price shifts is a world without submarkets. This contrasts strongly with a substitutability approach where submarkets can happily exist along a continuum. Indeed, one of the features of submarkets we might be most interested in is the extent to which boundaries are precipitous – whether forces that dominate are those that lead to discrete and specialised neighbourhoods, or those that lead to boundary gradation or aspatial submarkets.

In summary, the existence submarkets does not require discrete breaks in substitutability space (it is the fact dwelling substitutability is not constant that causes submarkets, not whether it is continuous). And discrete breaks in attribute prices imply neither discrete breaks in substitutability (see Criterion 1) nor evidence of submarkets. We therefore seek method of identifying submarkets that is not dependent on market imperfections or shifts in attribute prices.

Criterion 3: SEMs should be Robust to Unobserved Attribute Variation

Rationale: *If dwelling characteristics are spatially clustered, but not fully described in our hedonic data, then (a) observed differences in attribute prices can occur even when actual attribute prices are homogenous, and (b) observed spatially correlated errors in hedonic price regressions may simply reflect unobserved attribute heterogeneity. I.e. it is possible that (a)* $[\mathbf{z}_i^{\#} \neq \mathbf{z}_i] \Rightarrow \exists\ i, j$ *such that* $[\mathbf{P}(\mathbf{z}_i^{\#}) \neq \mathbf{P}(\mathbf{z}_j^{\#})] \wedge [\mathbf{P}(\mathbf{z}_i) = \mathbf{P}(\mathbf{z}_j)]$*; and (b)* $[\mathbf{z}_i^{\#} \neq \mathbf{z}_i] \Rightarrow \exists\ i, j$ *such that* $[C(e_i^{\#}, e_j^{\#}) > c^{*}] \wedge [C(e_i, e_j) \leq c^{*}]$, *where* $\mathbf{z}^{\#}$ *is the observed (as opposed* $\mathbf{z}$*, the actual) attribute bundle,* $e^{\#}$ *and e are the hedonic price equation residuals with and without attribute measurement errors, C is a measure of spatial correlation, and* c^{*} *is an accepted correlation threshold above which errors are deemed to be spatially correlated.*

Yes, dwellings are heterogeneous, but so are their attributes. If we are to correctly ascribe attribute price differences to submarket effects we must know all variations in attribute quality between dwellings. Measuring attribute *quantity* is also critical. That half a tank of petrol costs less than a full tank is no indication of discrepancy in price per unit. Likewise, apparent differences in price per room between tenements and modern flats may reflect unmeasured differences in room size (e.g. tenement rooms have higher ceilings) rather than submarket boundaries. Unfortunately, full information on the quality and quantity of every attribute of every dwelling is rarely, if ever, available; and the measurement errors that result will not be random but correlated with building type, which in turn is likely to be clustered across space. Boundaries derived from coefficient shifts in hedonic regressions may therefore be coterminous with the spatial pattern of measurement errors, rather than market segmentation. This further alters the interpretation of spatially autocorrelated errors in studies such as Tu *et al.* (2007).

Attribute measurement errors would not, however, affect measures of substitutability that are based on the price of the overall housing bundle. If two dwellings (or two blocks of dwellings) genuinely belong to the same submarket, one would expect the price of the overall housing bundle (or the average price of a dwelling bundle in each block of dwellings) to respond in a similar way to demand and supply shocks, irrespective of attribute prices. Focusing on the dynamics of the sale price of the entire housing bundle (which is generally measured with precision) rather than attribute prices (which are not) as the basis for submarket analysis is potentially a fruitful way for submarkets research to develop.

Criterion 4: SEMs should not impose or assume Convexity, Compactness, or Contiguity

Rationale: *If substitutability is granular (non-contiguous), non-convex or non-compact in Cartesian space, neither distance nor contiguity will adequately describe the spatiality of submarkets. I.e. the conditional probability of selling prices between properties as a function of substitutability,* $Prob[\mathbf{P}(\mathbf{z}_i)|\ \mathbf{P}(\mathbf{z}_j)] = f(\eta_{ij})$, *will be poorly explained by distance and/or contiguity because it is possible that,* (*a*) $\exists\ i,j$ *such that* $[d_{ij} > \tilde{d}] \wedge [\eta_{ij} > \tilde{\eta} \Rightarrow i,j \in S_k]$; (b) $\exists\ i,j$ *such that* $[d_{ij} \leq \tilde{d}] \wedge [\eta_{ij} \leq \tilde{\eta} \Rightarrow i,j \notin S_k]$; (c) $[v_{ij} = 0] \wedge [\eta_{ij} > \tilde{\eta} \Rightarrow i,j \in S_k]$; (d) $[v_{ij} = 1] \wedge [\eta_{ij} \leq \tilde{\eta} \Rightarrow i,j \notin S_k]$, *where* η_{ij} *is a measure of substitutability between i and j, and* $\tilde{\eta}$ *is the cut-off point beyond which i and j are considered close substitutes (and hence members of the same submarket),* d_{ij} *is the distance between i and j,* $\tilde{d}$ *is the cut-off point above which dwellings are considered distant, and* v_{ij} *is a binary variable equal to one if i and j are contiguous.*

A crucial factor that has tended to be overlooked in the debate over the degree to which submarkets are structural or spatial[7] is the qualifying effect on the role of distance of submarket *shape*. The effect of distance in determining substitutability will be complicated considerably if submarkets are elongated, or have holes (as in concentric circles). In such situations, dwellings can be far apart but still be in the same submarket. This has profound implications for the use of distance to approximate submarket effects and for the use of distance to capture the effect of proximity in spatial econometric models. If we use distance to define the spatial weights matrix when submarkets are non-convex then the errors observed will reflect a mixture of errors from attribute and amenity mis-measurement (which will be likely clustered across space – see above), and errors that arise from the failure to account for non-convex patterns of substitutability (which are also likely to be distributed non-randomly and nonlinearly in Cartesian space).[8] We therefore seek a methodology that will do justice to the complexity of submarket spatiality – reveal the shape of submarkets rather than imposing or ignoring it. Ideally, if we are interested in incorporating a spatial weights matrix, is should be based on *substitutability distance* rather than Euclidean, Manhattan Block, or Minskowski distance (see Anselin 1988, p. 17).

Surprisingly, the theoretical processes that fashion the geographical footprint of housing market areas has received little attention. This contrasts with the literature on theory of the firm, where the costs of transporting goods to and from the point of production to the point of consumption leads one to expect that "there would be forces at work to minimize total transportation costs" (Puu 2003, p.104). In turn, this creates a tendency to converge to some optimal market shape. Lösch (1940), for example, argued that the optimal shape of a market area for a single isolated firm would be circular. When there are many firms, the optimal shape of an individual market area is determined by a complex set subdivisions, of which the hexagon is the most compact optimal shape under a variety of conditions.[9]

It is beyond the scope of this paper to posit a general theory of submarket morphology but we can point to good reasons, along the lines of Schelling's (1971) seminal chequerboard model (see review by Meen and Meen, 2003), to believe that the sorting of

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[7] Some studies group by area (Straszheim 1975, Palm 1978), while others cluster dwellings by attributes or some other non-spatial criterion (Rothenberg *et al* 1991; see review by Watkins 2001). Most recent studies, however, acknowledge that there are both spatial and non-spatial drivers of submarkets and so some form of joint estimation is used (Bourassa *et al* 2003; Leishman 2009).

[8] A theoretical justification for including spatially lagged dependent variables in hedonic models is to capture the displaced demand, but displaced demand is much more likely to impact on dwellings that are *in the same submarket*. Buyers will switch to alternative dwellings that are close in substitutability space; not necessarily close in Euclidean space

[9] Christaller (1933) found some empirical support for the hexagonal market area in his study of firms in Southern Germany.

households across space will have a systematic component. Preference for racial or social homogeneity, for example, might lead us to expect compact, convex shapes, but there may be other factors (the cumulative history of residential planning decisions, access to employment, schooling, local amenities, radial and orbital transport links, heterogeneous preference for mix, etc.) that frustrate such processes. Therefore, the degree of compactness and convexity of a city's submarkets may tell us something of the potency of social sorting mechanisms relative to other forces that might mould more idiosyncratic spatial forms.

Note, in contrast, the implications of a significant minority who are indifferent to, or actually prefer, racial and social mix. Then there could be a large number of households that are happy to live along submarket boundaries, diluting the tendency for market forces to minimise the boundary length. Concentric rings of the access-space model are also highly non-convex sets in Cartesian space. In contrast, large cities may be "as much characterised by residential sectors as they were by residential rings" (Maclennan 1982, p.23). This is because "In the early phase of urban development, the most affluent and influential social and economic group were not sufficiently numerous to occupy a complete residential ring of the city. Instead, they tended to gather within a well-defined area or sector on one side of the city centre." (*ibid*). As the city develops, one might expect the city to be made up by a patchwork of residential enclaves, each with its own core and periphery. So, submarkets of the type described by Maclennan may be equally non-convex in Cartesian space, but made up of many sets of concentric circles centred on multiple cores, rather than a single sequence centred on the CBD. [10]

Other things being equal, Schelling-type processes would also lead one to expect submarkets to be made up of contiguous dwellings. Granularity may occur, however, when such spatial processes are weak or are broken down by countervailing effects of planning and heterogeneous preferences for mix, leading to dwellings from one submarket being scattered (in Cartesian space) among dwellings from another. Two issues arise at this point. First, the need to derive a method of submarket boundary identification that does not preclude granularity. Even if submarkets are spatial there is no reason to assume that all elements will be contiguous. Second, the extent to which elements of submarkets are contiguous or non-contiguous (granular) is itself of interest because it may reveal important spatial aspects of self-ordering processes and how these vary within and between cities.

[10] Note that many of these possibilities are at odds with the convexity restriction imposed in Clapp and Wang (2006).

Note that the foregoing discussion is not intended as a general critique of hedonic estimation – only that we should be highly sceptical about using HAPV as a way of understanding or deriving submarkets. Hedonic methods have many other applications, not least as a means of controlling for the mix of properties coming onto the market when computing measures of house price change. Indeed, use is made of this very feature in the empirical illustration below.

3. Deriving a Substitutability Approach to Submarkets

Early work on submarkets (Rapkin *et al.,* 1953) and Grigsby, 1963) drew directly on the concept of substitutability: "A housing market area is the physical area within which all dwelling units are linked together in a chain of substitution…" (Rapkin *et al*., 1953, pp. 9-10 quoted in Grigsby, 1963, pp. 33-34). It has been difficult, however, to operationalise this approach empirically. To measure the degree to which two goods are close substitutes, we might seek to estimate the cross price elasticity of demand (CPED). Unfortunately, CPED analysis requires estimating how the demand for one attribute bundle is affected by the selling price of another, but "Observed marginal hedonic prices … reveal little about underlying supply and demand functions" (Rosen, 1974, p. 50). Therefore, some alternative method of approximating CPED, based on prices of the entire housing bundle, seems the most promising way to proceed.

Rothenberg *et al.* (1991) made a concerted effort to measure substitutability but their approach relied heavily on the stability of hedonic coefficient estimates, which is problematic (see Maclennan 1982). The alternative to HAPV put forward in this paper attempts to exploit the dynamic nature of the market and make use of relationships between price changes (rather than price levels). Essentially, the Cross Price Elasticity of Price (CPEP) is proffered as a proxy for the Cross Price Elasticity of Demand (CPED), and hence of substitutability.

Proposition 1. *If demand and supply curves are well behaved (sloping downwards and upwards respectively), the cross price elasticity of price will have a strictly positive, one to one, relationship with the cross price elasticity of demand.*

Intuitively, the CPEP approach to substitutability can be understood as follows. Dwellings *i* and *j* are substitutes if a rise in the price of *j* leads to an increase in the demand for good *i;* hence, CPED > 0. Conversely, if *i* and *j* are complements, then CPED < 0. Now consider the following corollary. A rise in the price of *i* will cause a large increase in the demand for *j*, if *j* is a close substitute, and if the supply of houses is less than perfectly elastic, the short run effect of the increase in demand for *j* will be an increase in the price of *i*. That is, $\uparrow p_j \Rightarrow \uparrow Q_{Di} \Rightarrow \uparrow p_i$ (*cet par*).

The argument can be expressed more formally by considering the following equilibrium condition in the market for dwelling type *i*:

$$Q_{Si}(p_i, \boldsymbol{W}) - Q_{Di}(p_i, p_j, \mathbf{Z}) = 0 \qquad [4]$$

where **Z** and **W** are vectors of exogenous factors affecting demand Q_D and supply Q_S respectively, and p_i is price of the inseparable housing bundle *i*. By implicit differentiation of [4], the Cross Price Elasticity of Price is derived as:

$$\eta_{ij} = \left(\frac{dp_i}{dp_j}\right)\left(\frac{p_j}{p_i}\right) = \left(\frac{\partial Q_{Di} / \partial p_j}{(\partial Q_{Si} / \partial p_i) - (\partial Q_{Di} / \partial p_i)}\right)\left(\frac{p_j}{p_i}\right) \qquad [5]$$

Provided all prices are positive (p_i, p_j > 0), the demand curve for *i* is downward sloping ($\partial Q_{Di}/\partial p_i$ < 0), the supply is upward sloping ($\partial Q_{Si}/\partial p_i$> 0), and *i* and *j* are substitutes rather than complements ($\partial Q_{Di}/\partial p_j$ > 0), it is clear that CPEP will be positive.

Now compare [5] with the formula for CPED (derived again by implicit differentiation of [4]):

$$\varepsilon_{Q_{Di}, p_j} = \left(\frac{dQ_{Di}}{dp_j}\right)\left(\frac{p_j}{Q_{Di}}\right) = \left(\frac{\partial Q_{Di}}{\partial p_j}\right)\left(\frac{p_j}{Q_{Di}}\right) \qquad [6]$$

Again, provided prices and quantity are positive (p_i, Q_{Di} > 0), demand slopes downwards, and *i* and *j* are substitutes rather than complements ($\partial Q_{Di}/\partial p_j$ > 0), it is clear that CPED will also be positive. Rearranging [6] in terms of the numerator partial derivative we get $\partial Q_{Di}/\partial p_j$ = (Q_{Di} / p_j) $\eta_{QDi,pj}$. Substituting this expression into [5], we obtain CPEP as a function of CPED,

$$\eta_{ij} = \theta . \varepsilon_{Q_{D_i}, p_j}$$

where,

$$\theta = \frac{Q_{Di} / p_i}{(\partial Q_{Si} / \partial p_i) - (\partial Q_{Di} / \partial p_i)}$$

The numerator will always be positive, as will the denominator, so long as the demand and supply curves for dwelling *i* slope downward and upward respectively. It follows that the CPEP will be monotonically increasing in the CPED,

$$\frac{d\eta_{ij}}{d\varepsilon_{Q_{D_i}, p_j}} > 0,$$

and since the CPED is a measure of substitutability, it also follows that CPEP can be interpreted as a proxy. Crucially, however, CPEP does not require us to explicitly decompose the demand function. This is an important advantage because micro economic attempts to isolate housing demand rely heavily on hedonic estimation (e.g. Rothenberg *et al.* 1991; Ermisch *et al.* 1996). Instead, we can approximate η_{ij} using the slope coefficient from a regression of π_{ti}, the proportionate change over time in the price of dwelling (or block of dwellings) *i*, on π_j, the proportionate change over time in dwelling (or block of dwellings) *j*:

$$CPEP_{ij} = \eta_{ij} = \frac{dp_i / p_i}{dp_j / p_j} \approx \frac{\partial \pi_{ti}}{\partial \pi_{tj}}.$$

If $\eta_{ij} > 0$, then *i* and *j* are substitutes. CPEP increases with the level of substitutability to the point where $\eta_{ij} = 1$, which indicates that *i* and *j* are perfect substitutes and proportionate changes in the price of *i* are always matched by proportionate changes in the price of *j*. If $CPEP_{ij} < 0$ then *i* and *j* are complements. There is no obvious reason why $CPEP_{ij} > 1$ should occur other than as a result of market friction. For example, there may be contemporaneous overshoot of p_i in response to a change in p_j, possibly as a result of a lagged response to changes in p_j from an earlier period, or it may simply reflect idiosyncrasies in the transactions process (such as extreme bids – see Levin and Pryce 2007 and Smith *et al.* 2006), which can be counted as white noise. In the long run, and in the absence of market frictions, however, it is implausible that CPEP would be greater than unity, so $\max[E(\eta_{ij})] = 1$.

4. Using CPEP to Understand the Existence and Spatiality of Submarkets

Existence

CPEP leads to a natural test for the existence of submarkets. If CPEP = 1 for all pairs of dwellings, then all dwellings are perfect substitutes and there is no market segmentation:

$$\text{if } S_1 = \mathbf{M} \quad \text{then} \quad \eta^* = \max[E(CPEP_{ij})] = 1 \ \forall\ i,j \ , \ \text{where } i,j \in \mathbf{M}$$

We can represent the non-existence of submarkets in η_{ij}, d_{ij} space by a horizontal scattering of points all exactly equal to (or randomly scattered around) η^*, the value representing perfect substitutability. This scenario is depicted graphically in Figure 1 .

Figure 1 Single Unified Housing Market (S_1 = M)

Spatiality: the Effect of Distance

As discussed earlier, submarket classification methods are often distinguished as being either *Spatial* or *Non-Spatial*. The latter can, in fact, be constituted as an aggregation of the former: non-spatial submarkets can be defined as a higher-level grouping of spatial submarkets. To illustrate, define $S_k \subseteq \mathbf{S}$ as representing a spatial submarket such that elements of S_k have been allocated to this submarket using a cluster criterion that includes an explicit spatial component. Using a non-spatial criteria of defining submarkets leads to the disjoint grouping of spatial submarkets S_k into a smaller number of larger disjoint sets N_w,

Non-spatial submarkets: $N_1, N_2, \ldots N_W \subseteq \mathbf{M}$

where $W \leq K$. A particular collection of spatial submarkets into N_w, is defined by grouping together certain spatial submarkets, S_l and S_m, with average substitutability above some critical threshold $\tilde{\eta}$, where S_l and S_m are not necessarily contiguous:

$$N_w = \bigcup_k S_k \text{ for all pairs of submarkets } S_l \text{ and } S_m \text{ such that } \eta_{l,m} > \tilde{\eta}$$

The urban housing market then constitutes the union of mutually exclusive non-spatial groupings of spatial submarkets:

$$M = \bigcup_m N_m, \text{ where } N_{m=i} \cap N_{m=j} = \varnothing \quad \forall\, i \neq j \qquad [7]$$

The methodological implication is that, provided we can first identify the set of spatial submarkets, we can always test whether any non-contiguous pair of spatial submarkets actually belong to a common non-spatial submarket, N_w.[11] One could therefore start by identifying spatial submarkets even if one is ultimately interested in non-spatial submarkets. This rule becomes less useful, however, if all (or most) spatial submarkets are singletons (i.e. in a world where there is no spatial clustering by submarket) properties in different submarkets are randomly scattered in Cartesian space.

[11] A similar rationale is used by Clapp and Wang (2006) in the context of defining neighbourhoods.

It would be useful to have an overall measure of the spatiality of the entire submarket system. Using our price-dynamic approach to measuring substitutability, a global indicator of spatiality for an urban area is given by gradient ϕ of the relationship between η_{ij} and Euclidean distance d_{ij} between pairs of dwellings (i,j):

$$\phi = \partial \eta_{ij} / \partial f(d_{ij}) \qquad [8]$$

For a simple generic measure of the effect of distance on substitutability, one could assume CPEP to be approximately linear in logged distance: $\eta_{ij} = \alpha + \phi \ln d_{ij}$. If proximity is not an important aspect of substitutability, then one would expect η_{ij} to be unrelated to distance, resulting in a spherical scatter of d_{ij} (measured by $\phi = 0$) on d_{ij} as in Figure 2. On the other hand, if proximity is an important determinant of substitutability (due to access to the same amenities and disamenities, for example), then one would expect $CPEP_{ij}$ to decline with distance, most probably at a decreasing rate, illustrated in Figure 3.

Figure 2 Non-Spatial submarkets: "Spherical" Scatter of η_{ij} on d_{ij} ($\Rightarrow \phi = 0$)

Figure 3 Spatial submarkets: Downward sloping Scatter of η_{ij} on d_{ij} ($\Rightarrow \phi < 0$)

Why might we expect there to be a scatter, rather than a line, of points in η, d space? If strong substitutability occurs between distant dwellings because of elongated and non-convex shapes of spatial submarkets, then dwellings at the extreme ends of that submarket may be highly substitutable but far apart. Also, there may exist scattered clusters of substitutable bundles due to similar dwelling, neighbourhood and amenity combinations occurring at different points in the city (Rothenberg *et al.* 1991 p. 64). Thirdly, there may exist non-causal (i.e. coincidental) contemporaneous movements in distant pairs of inflation time series. Unless we can screen-out spurious correlations, CPEP will not provide a sufficient condition for submarkets, only a necessary condition. Consequently, when deriving submarkets from the substitutability measure, there may be an argument for including an explicit spatial component in as a practical means of screening out spurious correlations between price changes of distant dwellings, reinstating the sufficiency of CPEP, particularly since spatial submarkets can then be compiled into non-spatial ones as noted above.

Lattice of Substitution and the Shape of Submarkets

Having decided on a measure that allows us to gauge the substitutability between a given dwelling $i = a$ and all other dwellings $j \neq a$ in the urban area, we can conceive of this set of bilateral links for the dwelling $i=a$ as a *Lattice of Substitution,* $L_a = \{\eta_{ij}: i = a\}$. This set of relationships can be represented as a digraph, as depicted in Figure 4 where each dwelling represents a node and each cross price elasticity, η_{aj} , represents an edge. We can also think of the lattice as a surface in Cartesian space, plotted for a particular dwelling $i = a$, where the hills of this surface represent dwellings that are considered by the market to be close substitutes to a and the valleys represent dwellings that are not considered close substitutes to a. Note that a separate surface (or digraph) could be plotted for each and every dwelling (or block of dwellings) in the housing stock. The full set of lattices $\mathbf{L} = \{L_i: i = 1, 2, \ldots, V\}$ therefore describes the entirety of the substitutability set for an urban area.

Figure 4 Digraph for a First Order Substitution Lattice

While submarkets are essentially a discrete concept, they nevertheless offer a potentially useful way of summarising this set of multiple substitution lattices. The simplest derivation of a set submarkets is one that categorises a single *Lattice of Substitution* into groups with similar levels of substitution with respect to dwelling $i = 1$. We label this a *First Order Categorisation* (FOC) and is defined as follows:

$$S_1, S_2, \ldots S_s \subseteq \mathbf{M} = \{i: \text{cluster}(\eta_{1j}) \text{ where } \eta_{1j} \in L_1 = \{\eta_{ij}: i = 1\}\ \} \quad [10]$$

FOC is essentially a matter of identifying contour lines of substitutability with respect to dwelling $i=a$. *Second Order Categorisation* (SOC) entails clustering according to two substitution lattices, L_1 and L_2:

$$S_1, S_2, \ldots S_K \subseteq \mathbf{M} = \{i: \text{cluster}(\eta_{1j}, \eta_{2j}) \text{ where } \eta_{1j} \in L_1 \text{ and } \eta_{2j} \in L_2\} \qquad [11]$$

These two substitution lattices can be combined into a single *Second Order Substitution Lattice*, illustrated as a digraph below (Figure 5). We might choose randomly the dwelling that constitutes the basis for L_2, or we might be more judicious and deliberately select a dwelling that is not a close substitute to dwelling $i = 1$. *Third Order Categorisation* would involve clustering according to three substitution lattices L_1 , L_2 and L_3 (or a *Third Order Substitution Lattice*), and so on. Using more than one substitutability lattice offers a means of triangulating our results. Ideally, one would like to perform a V^{th} order Categorisation but this is likely to be computationally prohibitive.

Note that the clustering processes described above are entirely aspatial in that dwellings (or blocks of dwellings) are clustered in substitutability space not Cartesian space. This is important because, while the outcome of this process may well lead to systematic patterns in Cartesian space, any apparent spatiality of clustering outcome will not have been imposed by the method (because the clustering is being done in substitutability space) and therefore should reflect the true spatiality of housing substitution.

One might nevertheless seek to include an explicitly spatial component in how dwellings are grouped (as a means, for example, of reducing the effect of spurious correlations between distant dwellings). That is, we might seek to identify a set of *spatial* submarkets where spatial submarkets are defined as a partitioning of dwellings where distance (or some other measure of geographical proximity) plays an explicit role in clustering dwellings. Note that this definition does not impose continuous boundaries or preclude granularity. An example of how this might be achieved is given by the following clustering statement which includes Cartesian coordinates (x, y) of each dwelling j as additional grouping criteria:

$$S_1, S_2, \ldots S_s \subseteq \mathbf{M} = \{i: \text{cluster}(\eta_{1j}, \eta_{2j}, x_j, y_j) \text{ where } \eta_{1j} \in L_1 \text{ and } \eta_{2j} \in L_2\} \quad [12]$$

Figure 5 Digraph for a Second Order Substitution Lattice

5. Empirical Illustration

Depending on the scale of the total housing market being considered, the above method could be applied to cross elasticities between individual dwellings, or between blocks of dwellings, neighbourhoods or even larger areas provided the spatial units are small relative to the whole. To illustrate how the CPEP method might be applied to blocks of dwellings we use 33,680 GSPC realtor transactions in the Strathclyde region for the period 1999 to 2007. All sales are within 30km of Glasgow city centre, and have attribute information on individual dwellings, and x, y coordinates for the centroid of each dwelling block (which contain, on average, 15 dwellings)[12]. A set of time series for each of the **10,057** dwelling blocks (denoted as *i*) were constructed as follows:

(1) A Third Order Taylor Series approximation of the house price surface was estimated separately for each year. Extending the parlance of Fik *et al.* (2003)[13], these are called Time-Location Value Signatures (TLVS), estimated using flexible functional forms that include interactions between attributes, *x,y* coordinates for the dwelling block centroids, area dummies (based on a priori information on where likely shifts in the price surface may lie[14]), and quarter dummies for year's surface estimation. Insignificant variables and dummies were then eliminated using a stepwise procedure. Note that each TLVS was estimated independently for each year, allowing coefficients complete freedom to vary over years. Coefficients were also allowed to vary over space through interactions with *x,y* coordinates and area dummies. Dwelling type and attributes are included to control for the mix of properties selling in a particular dwelling block in a given time period. Quarterly time dummies are also included (and allowed to interact with dwelling type and location)

(2) Having estimated a TLVS for each year with quarterly slope and intercept dummies, an estimated price, P_i, can be given for each block of dwellings *i* in each time period (quarter), $P_i = TVLS_{i(t)}$. A series of *inflation* surfaces were estimated for each intervening time period by calculating the vertical distance for each *i* between each successive TLVS ($\Delta P_i = TVLS_{i,(t=2)} - TVLS_{i(t=1)}$), as illustrated in Figure 6, and then computing this as a proportionate change ($\pi_{i(t=2)} = \Delta P_i / TVLS_{i(t=1)}$).

Figure 6 Computing Price Change for Dwelling Block *i*

Having created a series of surfaces of annual inflation (one for each quarter since 2000 quarter 1), it was possible to extract a time series of the estimated constant quality price inflation series for each *i* (or, indeed, any point in the geographical space covered by the

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[12] By dwelling block we mean the "postcode unit" which is the highest resolution of spatial coding of our data available. Each postcode unit ("dwelling block") contains around 15 dwellings. Postcode units should not be confused with "postcode centroids" which contain around two thousand dwellings.

[13] Similarly, Clapp and Wang (2006) "control for large and medium scale variation with a polynomial lattitude and longitude and spatial dummy variables".

[14] Area dummies are based on realtor jurisdictions, and local authority areas (property taxes, in the form of Council Tax levies, and service provision, vary by local authority).

model), where *i* are the dwelling blocks were transactions are recorded in the data (i.e. actual residential locations). Inflation time series were therefore created for each centroid of the 10,057 dwelling block[15] within 30km of Glasgow. The adjusted R^2 results for all nine regressions were as follows: 0.73 (1999), 0.73 (2000), 0.76 (2001), 0.71 (2002), 0.63 (2003), 0.58 (2004), 0.61 (2005), 0.64 (2006), 0.71 (2007).

a. Existence of Housing Submarkets

Calculating $\phi = \partial\eta_{ij}/\partial f(d_{ij})$ is not a trivial exercise. If there are 10,057 blocks of dwellings, then there are 10,057 x 10,057 potential correlations between inflation time series, and 10,057 x 10,057 distances to be calculated. Including correlations/distances from *i* to itself, and those correlations/distances from *i* to *j* when d_{ij} has already been calculated, results in over one hundred million pairs of dwelling units, (*i*,*j*), for which we need to compute $\ln\eta_{ij}$ and d_{ij}. CPEP, η_{ij}, for a random selection of 100,000 pairs was calculated as the slope coefficient from regression of π_i on π_j where π_i is the annual constant quality price inflation time series for *i*. The values of η_{ij} and d_{ij} (distance) are plotted against each other for d_{ij} < 8km in Figure 7 (beyond 8km the slope of the line of best fit becomes horizontal). The graph indicates that not all dwelling units are perfect substitutes (the values of CPEP do not lie along the horizontal line of unity as in Figure 1).

Figure 7 Scatter plot of β coefficients sample of 100,000 regressions of CPEP$_{ij}$ on c_{ij}

Spatiality: the effect of distance

Given the downward sloping relationship between substitutability over short distances, there is *prima facie* evidence in Figure 7 of spatial submarkets: the value of ϕ for the system as a whole is negative, $\phi = \partial\eta_{ij}/\partial\ln d_{ij}$ = **-.017** (Robust CI = -.0179825, -.0164733; R^2 = 0.02, n = 100,000). The use of logged distance in computing ϕ was imposed as a simplification. When we run a linear spline regression of η_{ij} on d_{ij} we find that that the slope declines in absolute terms. Up to 1km the sllope = $-.45$ (robust CI = -.4879, -.418; Adj R-squared = 0.0785). That is, for every 1km increase in distance between dwellings, the cross price elasticity, η_{ij}, falls by 0.45 units. From 1km to <2km, $\partial\eta_{ij}/\partial\ln d_{ij} = -.35$ (robust CI = -.3691,

[15] This represents a much higher spatial resolution for a city wide analysis than previous UK research.

-.3323; Adj R-squared = 0.0785). From 2 to <4km, $\partial\eta_{ij}/\partial\ln d_{ij} = -.01$ (robust CI = -.0128, -.0037; Adj R-squared = 0.0785). Beyond 4km, the distance effect on substitutability becomes negligible, $\partial\eta_{ij}/\partial\ln d_{ij} \approx 0$ (robust CI = -.0050, -.0030; Adj R-squared = 0.0785).

Nevertheless, it is clear from the very low R^2 values associated with Figure 7 that the substitutability between dwelling blocks has a large non-spatial component – at least in terms of the capacity of simple Euclidean distances to capture spatiality (even in the spline regression, 92% of the variation in CPEP is due to factors other than Euclidean distance). This provides support for the non-spatial conception of submarkets (an important theme in Rothenberg *et al.* 1991) and an imperative to further explore the shape of submarkets – the existence of convexity, granularity and non-compactness renders distance an incomplete measure of the spatiality of submarkets. While beyond the scope of the current study, one might also consider other determinants of CPEP, such as differences in neighbourhood characteristics of *i* and *j* (such as racial and social composition, crime rates, schooling of the respective neighbourhoods), and dwelling characteristics, as these are all potentially important determinants of substitutability. Note that by incorporating these variables at the final stage of the analysis (rather than simply clustering observations by physical and geographical attributes) we are able to see how important each variable is in determining CPEP, enabling us to answer such questions as whether racial contiguity preference is an important driver of substitution. Note also that such analysis would be in difference form (difference in racial make-up between dwelling *i* and *j*) allowing one to better isolate the effect in question (as in conventional difference-in-difference approaches).

Spatiality: is there variation in SM shape and granularity?

The spatiality of housing substitutability is confirmed when we plot the substitution lattice for d_{120}, a randomly selected dwelling block (G11 5LP). First order categorisation (based on $S_1, S_2, \ldots S_K \subseteq \mathbf{M} = \{i\text{: cluster}(\eta_{120j})$ where $\eta_{120j} \in SL_1\}$ where η_{120j} is the Substitution Lattice, essentially amounts to drawing contour lines (Figure 8) in x, y, η_{120j} space to segment the market.

Figure 8 Substitutability Lattice for d_{120} Plotted as a Contour Map

In terms of allocating dwellings to discrete submarkets based on more than one Substitution Lattice, two further dwelling blocks were selected (9206 and 3247) as the basis for two further substitution lattices. These were chosen on the basis of (a) being a distant substitute of d_{120} ; (b) being physically distant in terms of location from d_{120}.

Two questions arise at this point: (1) *How many clusters (submarkets) should we opt for?* (Most cluster algorithms allow the user to specify.) In the context of a substitution lattice approach, we are clustering along a continuum and so the number of groups is otherwise arbitrary. And (2) *Should we include an explicit spatial component in the cluster algorithm?* Bourassa *et al.* (2003) maintain that "the appropriate definition of submarkets depends on the use to which they are put" (p.12). This is particularly true when it comes to addressing these two technical questions. Policy makers, for example, may seek to carve the city up into a few large contiguous submarkets, in which case, one would set the cluster algorithm to derive a handful of clusters and include an explicit spatial component (in order to encourage contiguity).

For purposes of creating area dummies for use in hedonic prediction accuracy, one is likely to prefer many clusters, derived using a spatial component, because the more spatially specific the dummies, the more likely one is to account for the unmeasured attribute and amenity variation (see Criterion 3 above). This is illustrated below Figure 9 which plots the Adjusted R^2 and log likelihood results of a series of grid searches over number of clusters for a variety of cluster functions ((i) FOC = First Order Categorisation as described above, (ii) SOC = Second Order Categorisation: $S_1, S_2, \ldots S_s \subseteq \mathbf{M} = \{i\text{: cluster}(\eta_{120j}, \eta_{9206j})$; (iii) TOC = Third Order Categorisation: $S_1, S_2, \ldots S_s \subseteq \mathbf{M} = \{i\text{: cluster}(\eta_{120j}, \eta_{9206j}, \eta_{3247j})$; and (iv) TOsC = Third Order Categorisation with explicit spatial component: $S_1, S_2, \ldots S_s \subseteq \mathbf{M} = \{i\text{: cluster}(\eta_{120j}, \eta_{9206j}, \eta_{3247j}, x, y))$.[16] Including an explicit spatial component helps reduce the effect of spurious correlation between price movements of distant dwellings. Note that improvement of hedonic prediction accuracy does not imply better understanding of submarkets – merely that one has done a better job of capturing unobserved measurement errors which may be useful if one is using hedonic regression for purposes of mass appraisal. Similar priorities apply if one is seeking to specify an alternative to the contiguity matrix in a spatial econometric regression – i.e. if one is attempting to use membership of the same submarket as the binary variable – since having a large number of spatial clusters will help capture unmeasured attribute and amenity variation. For purposes of price index calculation, we might seek the number of clusters and the type of clustering that minimises deviances between the six main index types listed in Hill and Melser (2008).

[16] Experiments using distance d_{ij} as the explicit spatial component were also attempted but this tended dominate the clustering algorithm and produce implausible concentric circles around *i*.

Figure 9 Adj R^2 and LL Diagnostics

However, if the goal is to identify the shape of submarkets – whether they congeal around a single set of concentric circles (following access/space trade-offs or orbital transport links for the city as a whole), or whether submarkets are idiosyncratic, non-compact, non-convex granular shapes (formed around local social and urban amenities), whether they are highly convex (driven by social segmentation and preference for homogeneity), whether submarkets are made up of contiguous or non-contiguous dwellings (Rothenberg *et al.* 1991 p. 64), or whether they are spatial at all (independent of Schelling-type processes), then one should probably avoid including an explicit spatial component in the cluster algorithm. This is because most cluster functions tend to impose convexity. Omitting the spatial component will not prevent the imposition of convexity in CPEP space but this will not cause the submarkets to be convex in Cartesian space as demonstrated in Figure 10 which shows the result of Second and Third Order Categorisation. We can see the effect of including a spatial component to the cluster function by comparing Figure 11 (which plots the results of a cluster function that includes x, y coordinates), with Figure 10. As for deciding on the number of clusters, this could be selected on the basis of dendrograms (which in this empirical example, tended to suggest clusters of between eight and ten submarkets, but do not always reveal a clear cut point) or with respect to the spatial scale of other variables of interest (such as racial segmentation).

What can we conclude from the submarket maps of Figures 10 and 11, which have been derived using a method that does not impose spatiality, contiguity or convexity in Cartesian space? First, they tell us that submarkets are a spatial phenomenon. While submarkets appear fragmented across space, the scattering is not aspatial, but grouped into distinct local sub-clusters. Second, we can see that the spatial effect is not a simple function of distance – there is evidence of non-convexity, non-compactness and non-contiguity (granularity). Third, there is no evidence of the concentric rings predicted by the standard urban economic model – if anything the clustering of dwellings is more radial than orbital.

Figure 10 Submarket Categorisation

Figure 11 Spatial Third Order Categorisation

3. Conclusion

This paper has sought to challenge conventional thinking on the definition and estimation of housing submarkets and provide a more robust theoretical basis for their existence. The goal has been to re-open and broaden the debate over submarkets by highlighting weaknesses in the existing empirical consensus and to develop a method based on cross price elasticities between individual blocks of dwellings. This approach is grounded in the notion of substitutability as the defining concept of submarket analysis and, in principle, meets the criteria set out in section 2. The Cross Price Elasticity of Price (CPEP) should be robust to transformative interaction effects and attribute measurement errors (because dwellings are not decomposed into attributes for purposes of determining substitutability; they are instead treated as inseparable bundles). CPEP does not dependent on discontinuity (because substitutability – and hence submarkets – can exist along a continuum) and does not impose or assume convexity, compactness or contiguity (dwellings, or blocks of dwellings, are clustered in substitutability space rather than Cartesian space, hence revealing, rather than imposing, the geographical pattern of market areas).

It is hoped that the CPEP method will open up new avenues of submarket research. For example, CPEP makes the determination of substitutability a researchable entity. To what extent is substitutability driven largely by physical attributes of dwellings and to what extent is it determined by neighbourhood mix and amenities? These questions are difficult to address in a hedonic attribute price approach because similarity or dissimilarity of observed attribute prices may have little bearing on substitutability.

CPEP may also help to define more appropriate areas for index calculation. Hicks (1939) asserted that, if a group of prices move in parallel, then the corresponding group of commodities can be treated as a single good (see Deaton and Muellbauer 1980, p.120-122). Hicks' *Composite Commodity Theorem* has proved fundamental to the calculation of price indices because, when the theorem holds, Paasche, Laspeyres, Fisher, Geometric Paasche, Geometric Laspeyres, and Törnqvist price index formulae "all give the same answer" (Hill and Melser, 2008, p.594). However, "when there is some variation in price relatives across products, the formulas diverge from each other" (*op cit* p.594). One could view CPEP as a means of identifying composite commodities – i.e. groups of substitutable properties with similar price movements,[17] so it should lead to submarket groupings that have a more meaningful basis for index calculation.

The paper also noted that substitution lattice may also constitute a more appropriate basis on which to derive a weights matrix for use in spatial econometric modelling because Euclidean distance matrices are problematic as a representation of the spatial relations between dwellings when submarkets are granular, non-compact and non-convex.

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[17] While there are instances in the literature of tests being done for the relationship between price trajectories across submarkets, this is usually done as a means of verifying previously defined submarket areas rather than as a means of deriving them from the data (e.g. Jones *et al* 2003).

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Figures

Figure 1 Single Unified Housing Market (S_1 = M)

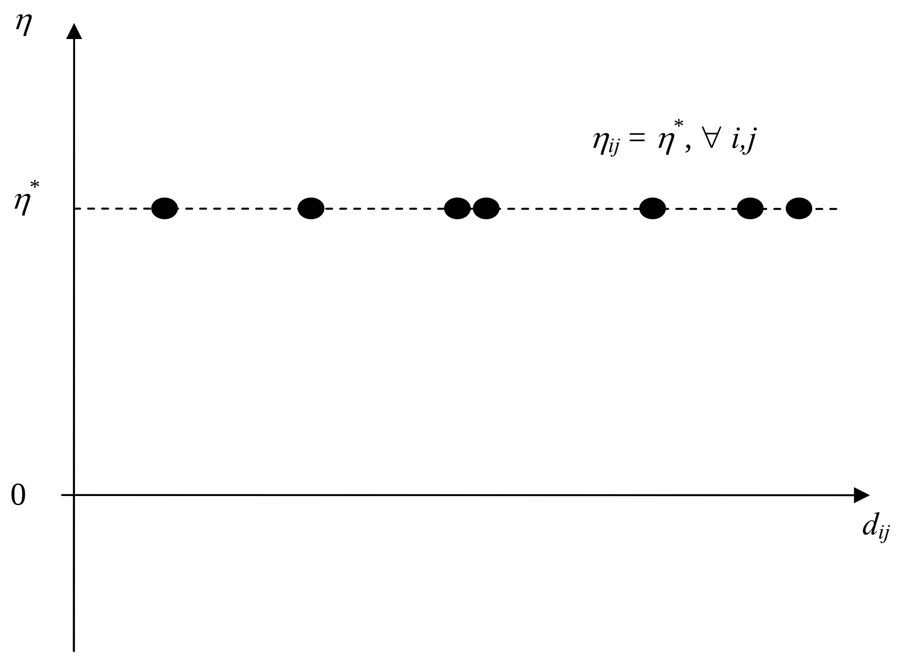


Figure 2 Non-Spatial submarkets: "Spherical" Scatter of η_{ij} on d_{ij} ($\Rightarrow \phi = 0$)

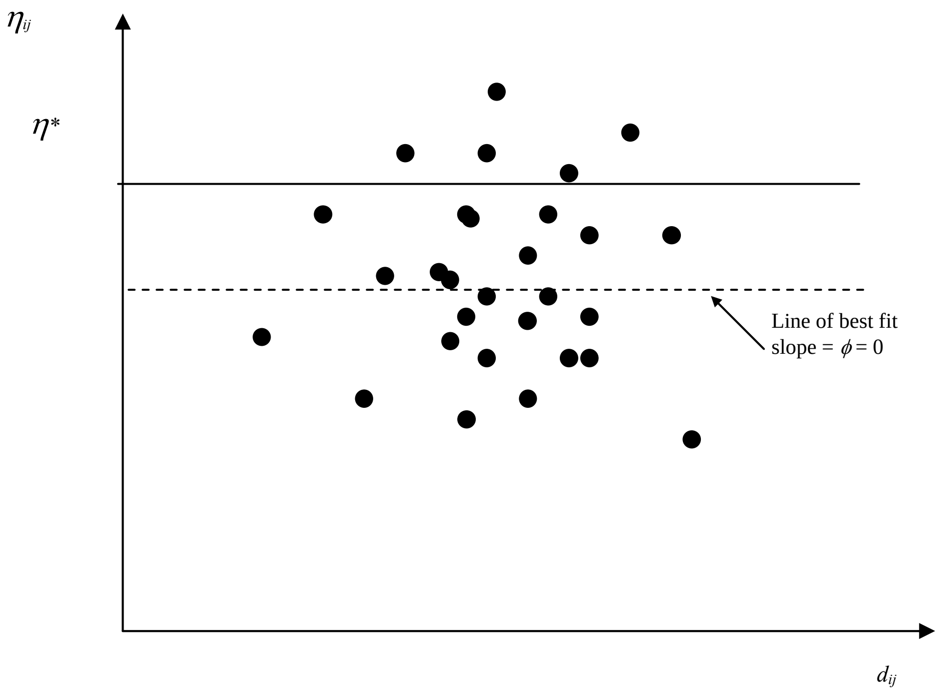


Figure 3 Spatial submarkets: Downward sloping Scatter of η_{ij} on d_{ij} ($\Rightarrow \phi < 0$)

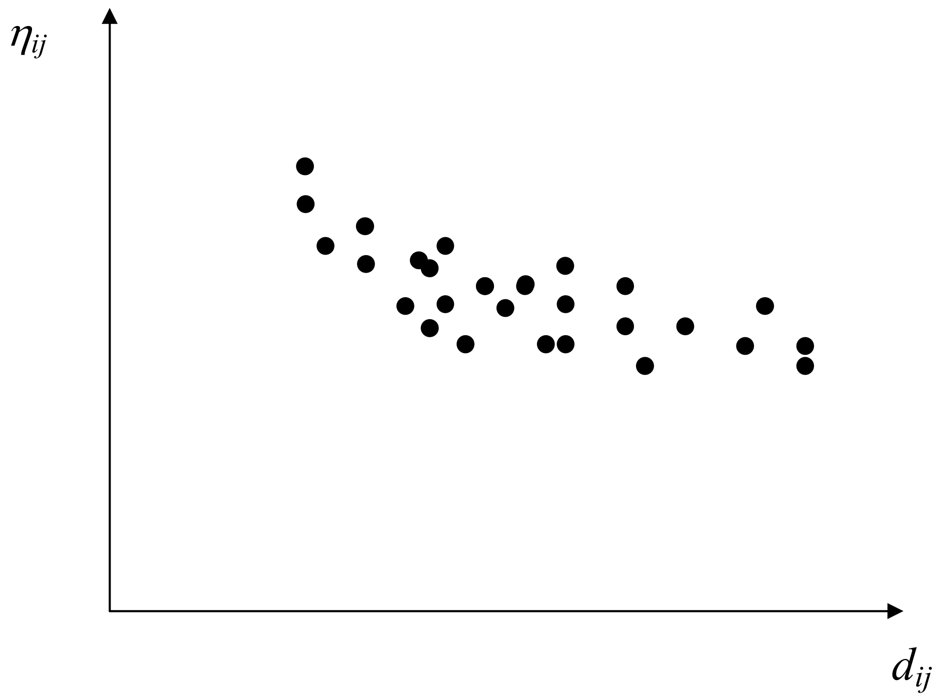


Figure 4 Digraph for a First Order Substitution Lattice

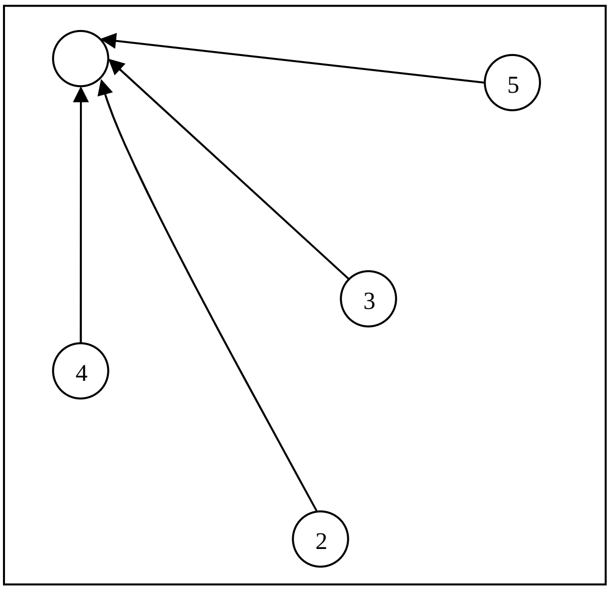


Figure 5 Digraph for a Second Order Substitution Lattice

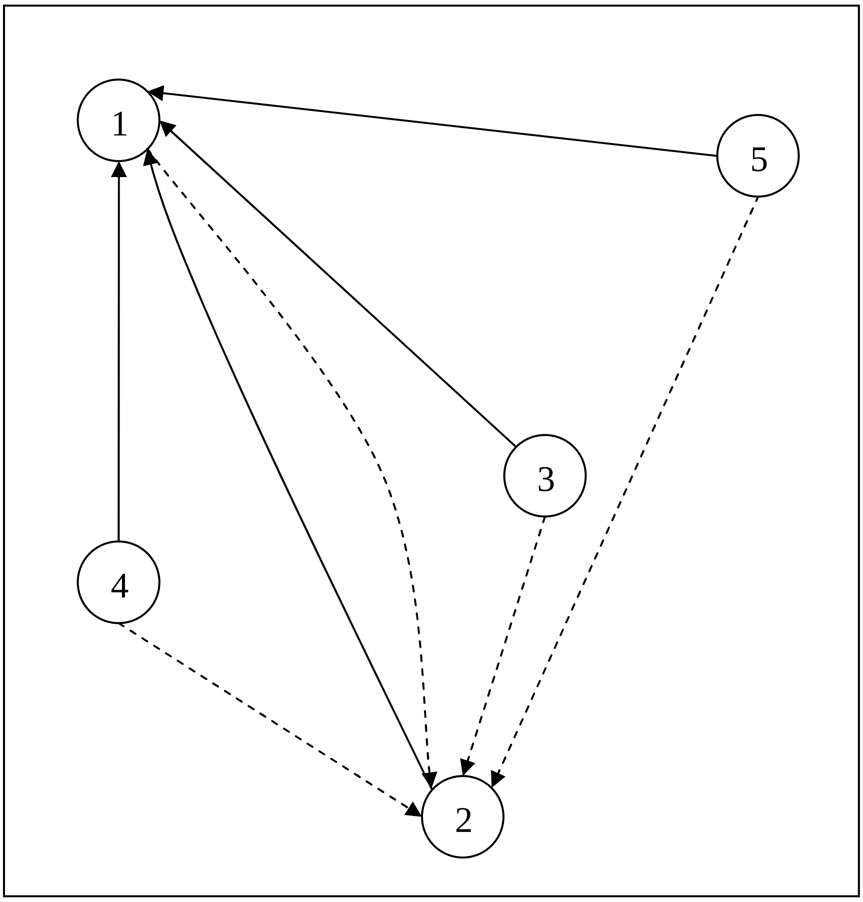


Figure 6 Computing Price Change for Dwelling Block *i*

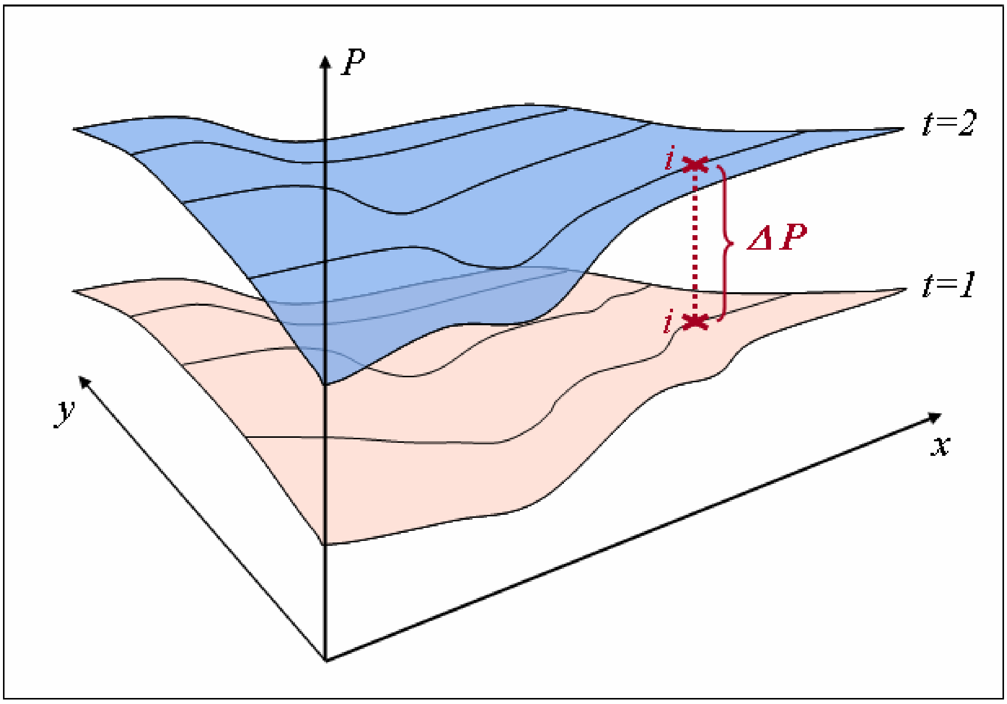


Figure 7 Scatter plot of β coefficients sample of 100,000 regressions of CPEP$_{ij}$ on c_{ij}

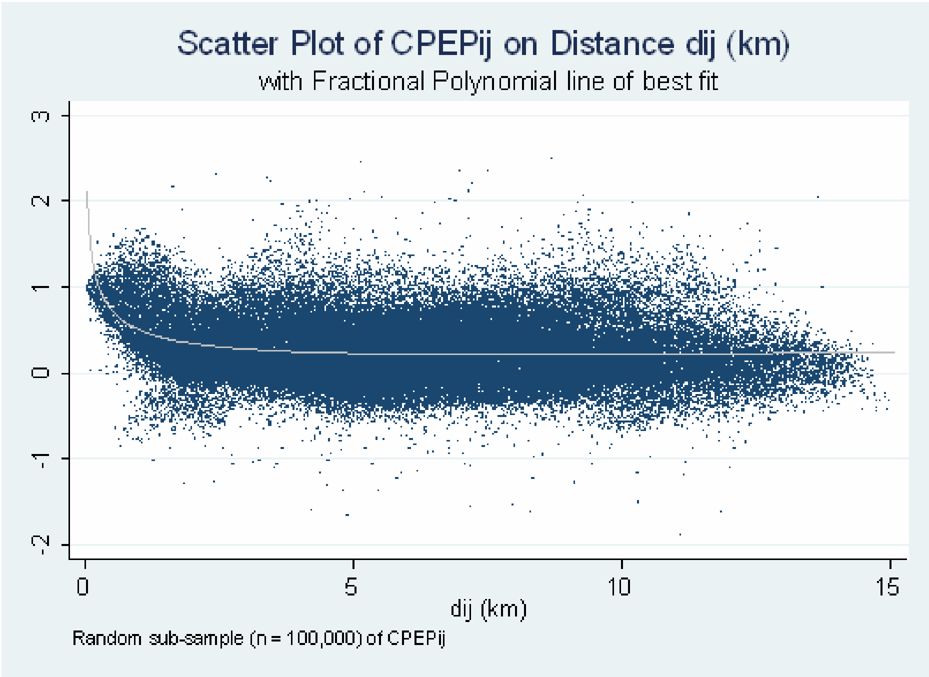
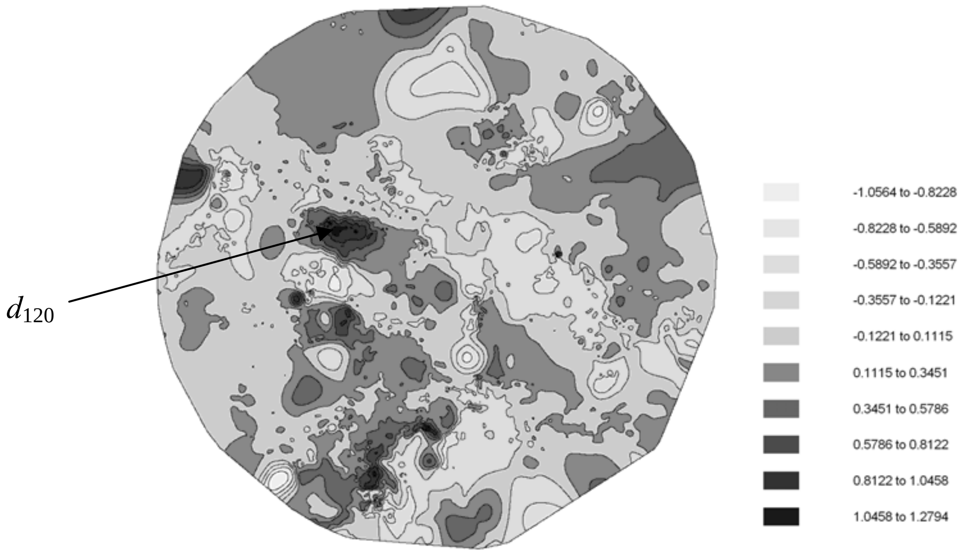


Figure 8 Substitutability Lattice for d_{120} Plotted as a Contour Map



Note: Plotted in Cartesian Space for Strathclyde (within a radius of 30km from Glasgow CBD)

Figure 9 Adj R^2 and LL Diagnostics

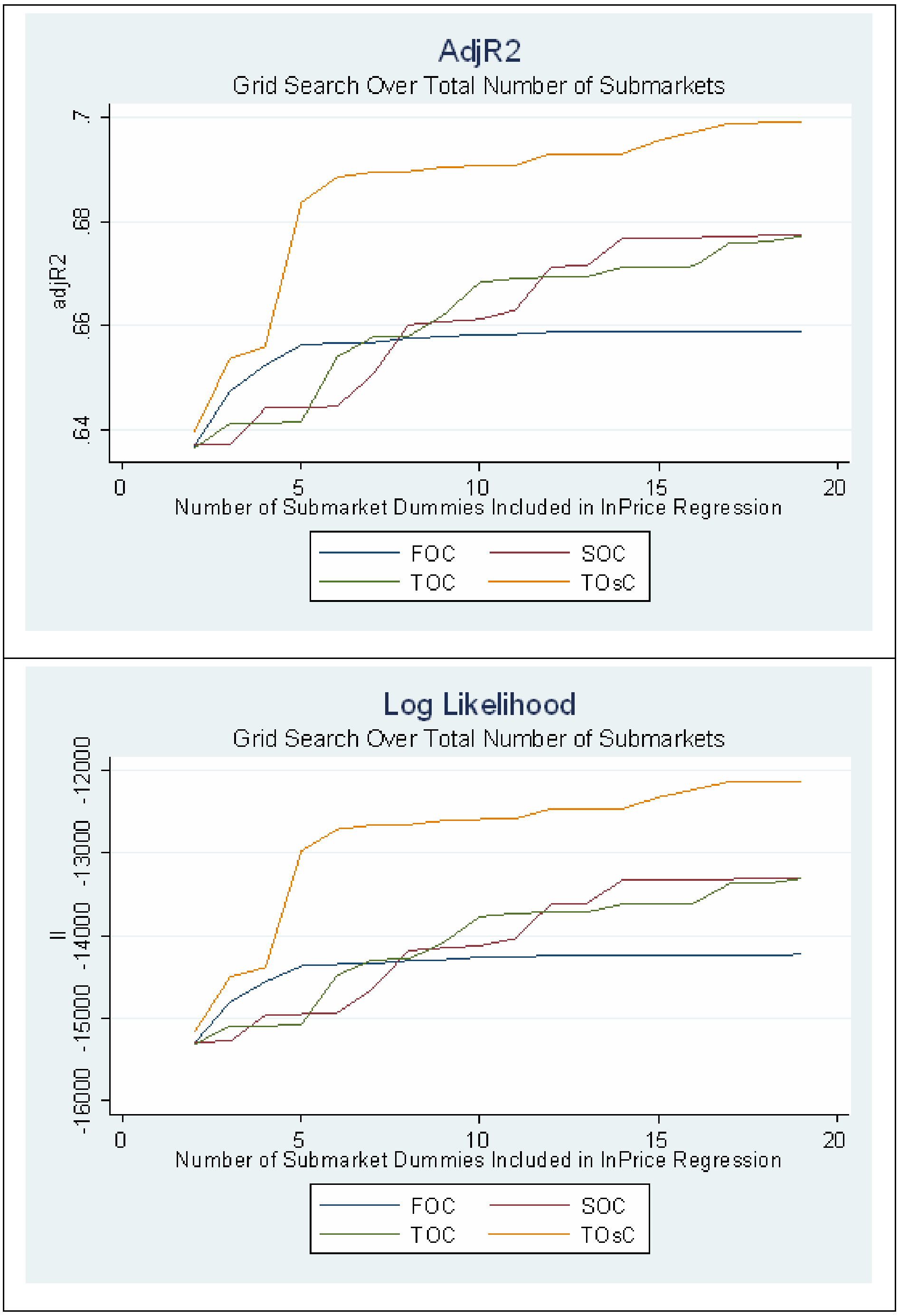


Figure 10 Submarket Categorisation

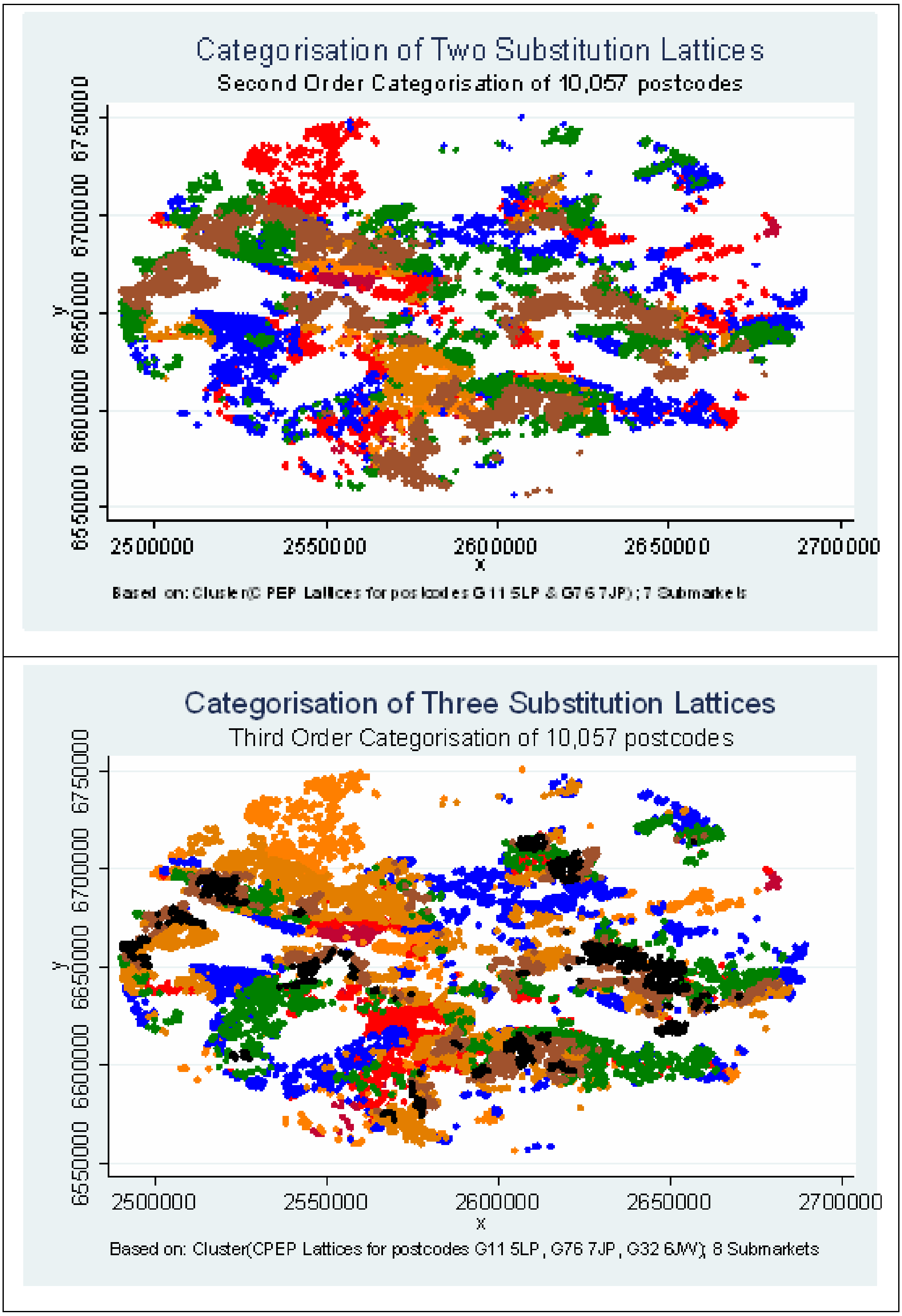


Figure 11 Spatial Third Order Categorisation

Using *x y* Coordinates