Abstract:
In this paper I consider the conditions under which risk pricing may not be advantageous to lenders by considering the implications of classificatory risk assessment for the S&W (Stiglitz and Weiss, 1981) model. The paper begins with a discussion of credit scoring/risk-pricing, followed by an overview of the credit rationing and risk assessment literatures. A discrete version of the S&W model is then developed and extended to include risk assessment. It is shown that risk assessment, and its corollary, differentiated interest rates, increases the consumer surplus captured by the lender. It is also shown, however, that pricing of loans based on risk categories can produce adverse selection, mitigating the ‘surplus-capture’ benefits of risk pricing for the lender. Although the borrowers screened-out by the introduction of risk-pricing would on average have higher default probabilities than those screened-in (because the worst of the good are better than the best of the bad), adverse selection can arise if the distribution of risks is such that the loss of “best of bad” risks raises the overall rate of default on the lenders’ loan portfolio. The paper also demonstrates that there is an absolute limit for optimal risk expenditure, and that S&W type credit rationing will not be precluded until risk assessment approaches this limit. The paper also presents an informal discussion of how the model may suggest incentives for niche marketing and how the model can be extended.

1 Introduction
Risk-pricing—the practice of charging a premium to higher risk customers—is common in many areas of finance because it has the obvious benefit of helping to ensure that the expected revenues from lending to a particular risk-type exceed the expected costs. Thus, higher risk car owners pay higher insurance premiums, and
less financially secure borrowers face wider interest rate spreads than their lower risk counterparts. For risk-pricing to be effective, however, the lender has to have a risk assessment procedure that accurately allocates borrowers to the relevant risk categories. The more refined the risk assessment procedure, the narrower the risk bands that lenders can define, and the more specific the interest rate that can be charged.

For most lenders, this process entails some form of ‘credit-scoring’ where each borrower is marked on a range of indicators thought to have some bearing on default risk. An overall score is then calculated and used to place the borrower in an appropriate risk category. Curiously, however, mortgage markets (particularly in the UK) have been slow to fully implement risk-pricing. Even though many mortgage lenders have been applying fairly sophisticated credit scoring techniques for a number of years, they have been reluctant to allow the results of the risk assessment to feed through into differentiated interest rates, choosing rather to use the information to ration credit by excluding the worst risks (according to Brown-Humes, 1997, three in ten people who apply for mortgages are turned away, for example).

What is the cause of the reluctance to price risk? The most obvious explanation is fear of bad publicity. Risk pricing in most mortgage markets would mean that a poorer individual in less stable employment would pay more for the same house than someone who is well off and enjoying secure employment. The implication? ‘Those who are able to pay the most are required to pay the least’ (Barnett, 1997, p.6). The social ramifications are heightened by the fact that employment and income brackets tend to fall within racial and gender lines. Hence, risk pricing in mortgage markets
could be perceived as a form of class, racial or sexual discrimination, as some of the negative publicity surrounding the issue has recently suggested (Barnett, 1997; Kempson, 1996; Herbert and Kempson, 1996).

However, there may be a more fundamental financial explanation for the lack of risk pricing in certain markets. This paper presents a simple theoretical model to demonstrate that, under certain conditions, risk pricing may cause adverse selection. It is already well established in the theoretical credit rationing literature that, in a pooled interest regime with asymmetric information, raising the rate of interest can cause adverse selection and this may provide lenders with an incentive to ration credit rather than raise the interest rate to clear the market. It is usually assumed, however, that moving to separating equilibria (i.e. different market rates for different risk types) would always be a desirable option for the lender if it is available since it will allow the lender to reduce the asymmetry of information and to charge more appropriate interest rates, reducing borrower surplus and the need for credit rationing. This paper shows, however, that if there is a spectrum of risks within each risk category identified for risk pricing (which is usually the case), then it is possible that for particular distributions of risk in the market, the move from a pooled interest rate to separate rates could have an adverse selection effect. These results are particularly pertinent to residential mortgage markets which are often paradoxically characterised by both high levels of risk assessment and limited use of risk pricing.

The remainder of the paper is structured as follows. First the credit rationing and risk assessment literatures are reviewed. A discrete version of the S&W (Stiglitz and Weiss, 1981) asymmetric information framework is then developed. In section 4, the
S&W framework is extended to include risk assessment and it is shown that differentiated interest rates increase the return on loans to a borrower of a particular risk type, but at the same time, the move to risk pricing has a screening effect which may not always be favourable. The paper also demonstrates that there is an absolute limit for optimal risk expenditure, and that there will be less scope for S&W type credit rationing as risk assessment approaches this limit. Section 5 presents a heuristic discussion of the implications of the results and the effect of relaxing certain assumptions.

2 Background Literature

To place the model in context, the credit rationing and risk assessment literatures (which have tended to develop quite separately) will now be considered. The larger of the two literatures is the former, primarily because of the important implications of credit rationing for a wide range of economic decisions. The impact on the macroeconomy, for example, has been discussed at length (Greenwald, B. and Stiglitz, J., 1993; Baachetta and Caminal 1996, Bernanke 1993, Bernanke et al 1994) following concerns that, inter alia, during ‘episodes such as the Great Depression, developments in credit markets seem to have amplified output fluctuations’ (Baachetta and Caminal, op cit, p.1; see also Bernanke, 1983), though systematic evidence on the link between financial factors and business cycles is still tentative (Bacchetta and Caminal, op cit).

Although credit rationing has been widely considered in the real estate literature (e.g. Haurin et al. 1996; Jones 1989, 1993; Linneman & Wachter 1989; Zorn 1989; Haurin
et al. 1997; Jones 1993; Ling & McGill 1998; Duca and Rosenthal 1994; Hendershott et al. 1997; Meen 1990a,b,c; Leece 1995, 2000) these papers are either entirely empirical or they tend to consider the consequences of credit rationing (such as on the tenure choice decision), rather than the causes. This is somewhat paradoxical, given that real estate credit markets (particularly residential mortgage lending) raise some particularly interesting questions for credit rationing theory. One particular characteristic of mortgage markets, for example, is the pervasive use of risk assessment\(^1\) (mainly because of the relatively large size and long term nature of most mortgage arrangements). Yet mortgage lenders appear to be more reluctant than most to apply risk premiums, even to borrowers who have already been ascribed credit scores. And even when different price categories are applied, some form of credit rationing usually persists. Although there exists a vast literature on credit rationing, and a growing real estate finance literature, to the author’s knowledge, the questions raised by the conjunction of credit rationing and risk assessment have yet to be addressed.

In fact, the theoretical literature on the causes of credit rationing remains relatively small given its significance. This is because, despite the long-recognised importance of credit rationing, finding a sound conceptual foundation for credit rationing equilibria has escaped economic theorists until relatively recently. The intractability has arisen from the surprising theoretical robustness of the traditional automatic adjustment mechanism of the market under the assumption of full-information. In most markets, a situation where supply does not equal demand constitutes a position of disequilibrium. Hence, ‘Conventional economic theory has traditionally viewed market clearing and market equilibrium as being one and the same’ (Clemenz, 1986,
p. 15). However, the equivalence of market clearing and equilibrium is not inevitable; it is merely the consequence of certain informational assumptions. If these assumptions are relaxed, particularly in the case of credit markets, a non-market clearing position (particularly excess demand for credit) is possible. Thus, equilibrium credit rationing is defined as occurring where there are no net forces in the system to bring about change to quantity or price, even though demand exceeds supply. For this type of credit rationing to be a theoretical possibility, there has to be some explanation of why it is not in the lenders best interests to raise the price of credit to clear the market.

A convincing rationale for equilibrium credit rationing did not really appear until the development of the theory of asymmetric information. The seminal work of Arrow (1964, 1968) and Akerlof (1970) showed how markets could radically deviate from their conventionally ascribed patterns of behaviour when the traditional assumption of complete information was relaxed. Arrow developed the principle-agent framework, and refined the notion of ‘moral hazard’: the possibility that where the preferences of principle and agent differ and where the principle’s knowledge of the agent’s behaviour is less than complete, the agent may be tempted to take actions which are sub-optimal for the principle. Akerlof’s contribution was to highlight the importance of adverse selection, which focused on, ‘the difficulty of distinguishing good quality from bad’ which Akerlof argued was ‘inherent in the business world’ and may ‘explain many economic institutions’ (1970, p. 500). In his example of the second hand car market, Akerlof showed how the buyer’s lack of information on quality may lead to sellers of high quality goods withdrawing from the market resulting in the fall in average quality of goods on sale (i.e. ‘adverse selection’). The first applications of
asymmetric information concepts to credit rationing were by Jaffee and Russell (1976), and Stiglitz and Weiss (1981). These applications showed how, unlike conventional markets, a rise in price has a deleterious effect on the quality of the lender’s loan portfolio, and thus provided a possible incentive for lenders not to raise the rate of interest to clear the market when there is excess demand. The theory remains relatively underdeveloped, however, for although there have been a number of extensions and adaptations to the S&W theory of credit rationing (Stiglitz and Weiss 1983; Bester 1985, 1987), models have yet to be developed which incorporate institutional factors such as risk assessment and credit insurance.

Risk Assessment

Part of the explanation for this omission is the accidental dichotomy between the development of the credit rationing and risk assessment literatures. Risk assessment studies for the most part have tended to fall into one of two categories: those that consider actual risk, and those that examine perceived risk. In analyses of actual risk, the focus is on borrower behaviour, and the dependent variable is usually a dichotomous one, reflecting the incidence of default. In the analysis of perceived risk, the focus is on lender behaviour and their attempts to model actual risk, and the dependent variable is some measure of perceived risk (such as spread over LIBOR or published risk ratings).

Because of data limitations, researchers have tended to focus on markets such as sovereign debt where both the actual risk of borrowers (such as Feder and Just, 1977; Alesina and Tabellini 1988; Lee 1991; Moghadam and Samavati 1991;) and perceived
risk (such as Feder and Just 1980; Calvo and Kaminsky, 1991; Seck, 1992; and Lee, 1993) can be analysed. Most of these papers are purely empirical, with little theoretical detail, and few acknowledge the possibility or implications of credit rationing (Seck 1992 is a notable exception). This is a major oversight, particularly for those papers measuring perceived risk using interest rate spreads since perceived lack of credit worthiness may be reflected in rationed credit rather than a larger interest rate spread. Given that the credit rationing literature has a more robust theoretical base, it makes sense for any attempt to link the two literatures to introduce risk assessment into a credit rationing model, rather than visa versa.

3 Basic Model

Purpose and form of the model

In the model below I attempt to extend the S&W model to include risk assessment. Because the focus of the model is on the impact of risk assessment on credit rationing and on the selection effect of interest rates, and not on the particular form that credit rationing may take, the representation of collateral is deliberately simple and the manifestation of credit rationing relatively general. The collateral term is not dropped altogether, however, for although some commercial real estate finance markets are non-recourse, most residential mortgage markets contain a recourse element (as do most commercial loans – Ooi, 2000) and so it is appropriate to retain some form of security in the model. It is worth noting at this point that in more sophisticated treatments of collateral, such as in the exploration of endogenous collateral by Bester (1987), it has been found that where lenders can vary collateral requirements, credit
rationing does not necessarily occur, even if there is asymmetric information (see Ooi, 2000, for a full discussion of the role of collateral in real estate). However, in most real estate lending situations the requirements of the investment project tend to determine the proportion of debt financing and not visa versa, and so one could argue that the variation of the collateral requirement by lenders itself amounts to a form of credit rationing. Indeed, this has usually been the view of real estate researchers who have tended to classify what Bester calls ‘endogenous collateral’ as LTV credit rationing (in Hendershott et al 1997, for example, credit rationing is represented in two ways: as a repayment to income constraint, and as a LTV limit).

It is also worth noting that the model developed below is very much in the asymmetric information tradition and so many of the full information/efficient capital market results do not apply. Note, for example, that lenders do not know the risk of individual borrowers (only the distribution of all risks and the category into which borrowers fall), and so portfolio decisions of the kind examined in the Capital-Asset Pricing Model are not directly relevant. Thus, models in the asymmetric information tradition (into which the current paper falls), such as Bester (1985, 1987) tend not consider the impact of portfolio size on overall risk (they effectively assume a large number of loan applicants), neither do they necessarily adopt a rate of return approach (see Hirschleifer and Riley 1995 for an elucidation of the differences between the full information and asymmetric information traditions in finance theory). Note, however, that the key findings of the paper (propositions 1 to 4) would still hold if we presented the model in terms of the rate of return since they are either independent of portfolio size (propositions 4 and 5), or based on average borrower risk within particular risk categories (propositions 1, 2, and 3) which would still be relevant for a given portfolio.
size (our concern is primarily with lender decisions to assess risk and pool interest rates rather than overall portfolio size).

**Initial Assumptions**

Consider a credit market with $n$ types of risk neutral entrepreneur (investor) $i$, where $i \in I$, and $I = (1, 2, ... n)$, each with the opportunity to invest in a project requiring a fixed amount of fixed capital, $K$. Banks in turn demand fixed collateral $C$ (which could be interpreted as the equity required on the loan), and charge interest rate $r$ on each loan. For simplicity we assume $C$ to be fixed in proportion to $K$. This is equivalent to saying that in the event of default, borrowers lose their fixed equity stake. Investor $i$’s project succeeds with probability $p_i$ yielding the positive return $R^i_i$; and fails with probability $(1-p_i)$ yielding zero return, where higher risk projects receive a higher return: $1 > p_1 > p_2 > ... p_n > 0$ and $K < R^1_i < R^2_i < .... < R_n$ (where the increments of $p_i$ and $R^i_i$ are proportionately of similar magnitudes).

It is assumed that lenders are risk averse, which is consistent with the characterisation of mortgage lenders in the Introduction as relatively cautious institutions, reluctant to introduce risk pricing for fear of bad publicity or some adverse underlying financial outcome. (Nevertheless, the results of the model are not contingent on the risk aversion assumption, since risk neutral lenders would face the same selection implications of their price setting decisions as those explored below). Lenders know the distribution of $R^i_i$ and the distribution of $N_i$, the number of loans made to risk type $I$, but they do not know the default probability of any individual loan applicant. 

*Ceteris paribus* the corollary is that, given the total number of loan applicants, the lender will be able to estimate the numbers of each risk type that have applied given
they will reflect the distribution of risks in the market as a whole and/or the
distribution of risks on its loan books in previous periods. The number of each risk
type in the market is large, as is the number of applications faced by each lender. It is
further assumed that the interest charged on deposits is unrelated to the terms of the
loan.

**Borrowers**

The elementary objective function of borrowers is given by \( \max [R_i^t - (1+r)K, -C] \) and
expected returns are given by,

\[
\pi^b_i = p_i[R_i^t(1+r)K] - (1-p_i)C. \tag{1}
\]

It is assumed that the entrepreneur of type \( i \) only takes out the loan if,

\[
\pi^b_i \geq 0. \tag{2}
\]

Thus, a necessary condition for an offer of a loan to be accepted, is that the return if
the housing investment is successful has to be greater than the total repayment costs:
\( R_i^t > (1+r)K \). This is obvious from equations [1] and [2] which imply that, \( p_i[R_i^t -
(1+r)K] \geq (1-p_i)C \), yielding the necessary and sufficient condition,

\[
\Rightarrow p_i \geq \frac{C}{R_i^t - (1+r)K + C}. \tag{3}
\]

Since \( 0 < p_i < 1 \), it follows that \( R_i^t > (1+r)K \). The number of loans demanded by risk
type \( i \) is thus given by

\[
N^D_i = \begin{cases} 
N^T_i & \text{if } \pi^b_i \geq 0, \\
0 & \text{if } \pi^b_i < 0,
\end{cases}
\]

where \( N^T_i \) is the total number of firms of type \( i \).
It can be shown that raising the rate of interest causes adverse selection when there is no risk assessment. First assume that, for a given interest rate \( r \), there is a threshold success probability \( p_{i#} \) (i.e. threshold type of investor) such that the entrepreneur borrows from the bank if and only if \( p_i \leq p_{i#} \) (i.e. \( i \geq i_# \)) where \( p_{i#}(r) < 0 \) (\( i_# \) is positively related to \( r \)).

The proof can be shown by contradiction. First note that \( p_{i#} \) is given where the borrower just breaks even. For \( \pi_i^b = 0 \) the weak inequality [3] becomes an equation,

\[
p_{i#} = \frac{C}{R_s^i - (1+r)K + C}, \quad [3.1]
\]

where \( R_s^i \) is the return if successful associated with \( p_{i#} \), given the fixed relationship between \( R^s \) and \( p \). Given that the loan is only applied for if \( R_s^i \geq (1+r)K \), it follows that higher rates of interest will raise \( R_s^i \) because of the negative relationship between \( R_s^i \) and \( i \),

\[
R_s^i(r + \varepsilon) > R_s^i(r), \quad \text{where } \varepsilon > 0,
\]

\[
\Rightarrow i_#(r + \varepsilon) > j_#(r). \quad [4]
\]

In other words, lower risks will not apply for a loan when \( r \) increases because it is not worth their while given the lower return on lower risk projects, and the greater cost of repayment when \( r \) increases. Now assume that \( p_{i#} \) is not strictly decreasing in \( r \),

\[
p_{i#}(r + \varepsilon) \geq p_{i#}(r) \Rightarrow i_#(r + \varepsilon) \leq i_#(r), \quad \text{which contradicts [4].}
\]
Lenders

Competitive lenders know the distribution of \( p_i \) and \( R_i^\tau \), and so know the value of \( p_{n,b} \), but cannot identify the \( p_i \) of a particular loan applicant. Lenders are risk averse and wish to maximise \( U \) where

\[
U = \sum_{i=1}^{n} u_i \quad [5]
\]

and \( u_i \) is the utility obtained from loans to borrowers of type \( i \). Banks finance their credit offers by funds from deposits. If \( \theta \) is the interest paid on deposits, the bank’s utility of net profits on a loan to investor \( i \) is given by,

\[
u_i := N_i [p_i u_i (1+r) K - (1+\theta) K] + (1-p_i) u_i \{ C - (1+\theta) K \}. \quad [6]
\]

Banks will only lend to borrowers where \( u_i > 0 \).

We can now also show how equilibrium credit rationing is possible if the lenders are imperfectly informed concerning \( p_i \). Credit rationing is defined as a situation where \( N_i < N_i^D \), where \( N_i^D \) is the number of loans demanded from risk type \( i \) (where \( N_i^D = N_i^T \) or 0). This implies that \( \exists i \) such that \( \pi_i^b(r) \geq 0 \) and \( N_i < N_i^T \). Such rationing can be sustained in equilibrium provided \( \exists i \) such that,

\[
U(r + \varepsilon) < U(r) \quad \text{and} \quad N_i < N_i^D
\]

where \( \varepsilon \in \mathbb{R}^+ \). Thus, raising \( r \) in certain circumstances will reduce overall lender utility for the lender, and so allow for the possibility that raising the interest rate will not be optimal, even when there is excess demand for credit. Proof can be established by contradiction. Suppose \( U(r + \varepsilon) > U(r) \) for some \( \varepsilon, p_i, N_i \) and \( r \), then,

\[
\Rightarrow \sum_{i=1}^{n} u_i (r + \varepsilon) > \sum_{i=1}^{n} u_i (r)
\]
which implies that one of two possibilities must always be true when \( r \) rises: either utility per loan type does not diminish when \( r \) rises and higher interest rates always imply a lower threshold risk group \( i_\# \); or utility per loan type is strictly greater when \( r \) rises, but the threshold risk group is weakly lower,

\[
\Rightarrow \left( u_i(r+\epsilon) \geq u_i(r) \text{ and } i_\#(r+\epsilon) < i_\#(r) \right) \text{ or } \left( u_i(r+\epsilon) > u_i(r) \text{ and } i_\#(r+\epsilon) \leq i_\#(r) \right),
\]

both of which contradict [4]. So for credit rationing to be precluded it has to be shown, not only that \( n - i_\# \) (i.e. the number of risk groups which the bank can sum across, remains the same or increases), but also that the utility to the lender of each loan to type \( i \) always increases. However, because lower risk groups are less likely to make a profit when interest rates rise, the bank will effectively screen out lower risk borrowers (\( i_\# \) always rises when \( r \) rises). Also, it should be noted that the bank gains more utility from lending to good risks cet par and so its average utility per loan type diminishes when good risks are screened out. The preference for good risks (cet par) can be shown by contradiction: if \( u_i > u_k \) and \( N_i = N_k \), where \( i > k \), then,

\[
p_i u_i \{ (1+r)K - (1+\theta)K \} + (1-p_i)u_i \{ C - (1+\theta)K \} > p_k u_k \{ (1+r)K - (1+\theta)K \} + (1-p_k)u_k \{ C - (1+\theta)K \},
\]

\[
\Rightarrow p_i > p_k ,
\]

which contradicts the ordering of probabilities. Thus the increased utility from raising \( r \) (due to the greater gross interest) has to be balanced against the lost utility from screening out good risks and the riskier loan portfolio that it implies. Notice that the bank will not always ration credit, but the above, first put forward by Stiglitz & Weiss op cit in the context of risk neutral banks, shows how credit rationing is not precluded under asymmetric information.
4 Risk Assessment

Investing in risk assessment allows the bank to distinguish between \( v^* \) risk groups amongst borrowers where \( v^* \) takes only positive integer values: \( v^* \in [1, \infty] \). The interval to which the lender can allocate borrower type \( i \) following the assessment of risk is given by, 
\[
P_v = \{ p_i : p_r \leq p_i < p_{r+1} \} \quad \text{where} \quad p_v = v/v^* \quad \text{and} \quad v \in V; \quad V = \{0, 1, 2, ..., v\}, \]
\( v = v^*-1 \); and \( P = \{ P_v : v \in V \} \). Qualities of \( P \) include (1) \( \forall v : P_v \subset P \); (2) \( v_1 \neq v_2 \Rightarrow P_{v_1} \cap P_{v_2} = \emptyset \); and (3) \( \bigcup_{v} P_v = P \). In other words, every point of \( P \) belongs to one and only one \( P_v \) (each subset \( P_v \) of \( P \) is therefore disjoint), and so the family of sets \( P \) is a partition. Risk assessment is “true” in the sense that borrowers are always correctly associated with the appropriate partition of \( P \). Since the bank knows the risk interval to which each potential borrower belongs, it is not possible for borrowers at the lower end of each interval, who may be faced with a rate of interest that makes borrowing unattractive (i.e. \( R^i < (1+r_i)K \)) to surreptitiously make their way into the lower category. Thus, borrower type \( i \) cannot dupe the risk-assessing lender into believing that he/she is anything other than \( \{ i : p_r \leq p_i < p_{r+1} \} \).

4.1 Costless Risk Assessment

For a given level of risk assessment, the lender aims to max \( \bar{U} \), where:
\[
\bar{U} = \Sigma \tilde{u}_v, \quad \text{[7]} 
\]
\( \tilde{u}_v \) is utility gained from a particular risk interval, and \( P_v \), which the bank can identify. This utility will comprise the sum of utilities from loans to all borrowers relevant to that risk interval, ranging from the highest risks admitted (determined by risk assessment) to the lowest, defined either by the upper bound of \( P_v \), or by the risk group that just breaks even given \( r_v \), whichever is the greatest. Note that, for each
identified interval, there will be a different interest rate and so there will be a different associated threshold success probability \( p_{\text{i}\#v} \) and associated \( i_{\#v} \), where \( p_{\text{i}\#v} = p_{\text{i}\#m}(r_v) \), \( p'_{\text{i}\#m}(r_v) < 0 \) and \( i_{\#v} = i_{\#m}(r_v) \), \( i'_{\#m}(r_v) > 0 \). Thus,

\[
\tilde{u}_v = \sum_{\max(i_v, i_{\#v})}^{i_v} N_v \left[ p_u \{(1+r)K - (1+\theta)K\} + (1-p_u)\{C - (1+\theta)K\}\right],
\]

where \( i_v \) is defined as \{i: \( p_i = \max(p_l < p_{v+1}) \}\), the lowest \( i \) admitted in \( P_v \); \( i_v \) is the highest \( i \) admitted in \( P_v \) defined as \{i: \( p_i = p_v \)\}; and \( i_{\#v} \) is the threshold risk group who will still find it profitable to apply for a loan given \( r_v \) (all \( i < i_{\#} \) will not apply). Note also that for each risk type there is a threshold interest rate \( r_{\#v} \) above which investors will not apply for a loan, and this is obtained by solving for \( r_v \) in the equation for \( p_{\text{i}\#v} \), which is derived in a similar way to \( p_{\text{i}\#} \):

\[
p_{\text{i}\#v} = \frac{C}{R_{\#v}^v - (1+r_v)K + C}.
\]

Borrowers will only apply for a loan if \( R_{\#v}^v > (1+r_v)K \).

If the bank sets \( r_v \) such that the threshold success probability is greater than or equal to the upper bound of \( P_v \),

\[
r_v = \{r_v: p_{\#v} \geq p_{v+1}\} = \{r_v: i_{\#v} \geq i_{*v}\},
\]

then all risk types in the interval \( P_v \) will apply for a loan because,

\[
R_{\#v}^v > (1+r_v)K, \forall i \in P_v.
\]

The number of loans made to investors in \( P_v \) will be \( N_v \) where \( N_v = \sum_{i_v}^{i} N_i \). In general \( N_v \) is given by

\[
N_v = \sum_{\max(i_v, i_{\#v})}^{i_v} N_i.
\]

Note that there is no incentive to set \( r_v \) below that which produces \( i_{\#v} = i_{*v} \).
\[ r_v = \{ r_v : i_{\#v} < i^* \} , \]
since the bank would lose revenue on each loan without gaining extra (low risk) loan applicants.

On the other hand, if the bank sets \( r_v \) such that the threshold success probability is less than the lower bound of \( P_v \),

\[ r_v = \{ r_v : p_{\#v} < p_v \} = \{ r_v : i_{\#v} > i_v \} , \]
then no risk types in \( P_v \) apply, and \( N_v = 0 \). Since the lender knows \( i^*_v \) and \( i_v \), and hence the associated probabilities and returns \( (p_1, p_2, R_1, R_2) \), it can compute the interest rates in each \( P_v \) required to achieve \( p_{1,v} \) and \( p_{2,v} \). Thus, the profit maximising bank will always set \( r_v \) such that \( r_{\min} \leq r_v \leq r_{\max} \), where \( r_{\min} = \{ r_v : i_{\#v} = i_v \} \), and \( r_{\max} = \{ r_v : i_{\#v} = i^*_v \} \). (NB: \( i^*_v < i_v \)).

To recap, summation is across all investors at least as risky as the threshold risk, \( i_{\#v} \) (determined by \( r_v \)), but less risky than the lower success probability bound \( p_v \). If \( r_v \) is set such that \( i_{\#v} > i_v \) then there will be zero loan applicants from the range \( P_v \). If \( r_v \) is set such that \( i_{\#v} \leq i^*_v \) then all risks in the range \( P_v \) will apply.

**Proposition 1:** Increasing risk assessment will always increase the return on loans to a borrower of particular risk type.

**Proof:**
Increasing risk assessment allows the bank to obtain some of the surplus previously attributed to borrowers because it allows the bank to charge a greater number of differentiated interest rates. This inevitably means that borrowers (for whom investment is still profitable in the state of greater risk assessment) that enjoyed a
large difference between their reservation interest rate, $r_{\text{res}}$, and the actual interest rate, $r_v$, will, under a regime of greater risk assessment, be faced with an interest rate that is closer to their reservation rate. Let $S$ be the total borrower surplus for all $P_v \in P$,

$$S = \sum_{v} s_v,$$

where,

$$s_v = \sum_{i}^{\text{iv}} (r_{vi} - r_v)N_i,$$

Raising $v^*$ results in a greater number of subsets of $P$, resulting in narrower intervals for each interest rate, $r_{v\text{min}} \leq r_v \leq r_{v\text{max}}$, and this will cause the average consumer surplus $s_v/N_v$ in each identified risk interval to fall. This means that for every loan made, the bank is receiving a greater return.

**Proposition 2:** Increased price differentiation produces favourable selection if $N_i$ is uniformly distributed, producing an overall utility gain for the lender, or monotonically increasing across $i$.

**Proof:**

Assume, for a moment, that risk assessment allows the lender to classify borrowers into two groups: a high risk band and a low risk band, with two corresponding interest rates. Assuming there are still many risk types within each of the two identifiable bands, some of the borrowers whose threshold interest rate was below the single pooled interest rate, will now be at the upper end of the low risk band, and find that the rate of interest they are offered is below their threshold rate. In contrast, some of those borrowers whose threshold interest rate was previously above the single pooled rate (and so willing to accept the loan offer) will now fall into the lower end of the high risk band and so be screened out by the new interest rate. However, the
borrowers falling into the lower end of the high risk band (and now priced out of the market) are more risky than those falling into the upper end of the low risk band (priced into the market by differentiated interest rates). Favourable selection occurs because the worst of the good are better than the best of the bad.

This is demonstrated in Figure 1 below, where a greater number of identifiable risk categories will result in some borrowers being priced out of the market, as well as others now being priced ‘into’ the market. The horizontal axis depicts the spectrum of threshold interest rates across $i$, given that each $i$ has a unique threshold interest rate, above which it is not worthwhile investing. Super-imposed onto the axes are the interest rates actually charged, denoted by $r$ under no risk assessment, and $r_1$ and $r_2$ following risk assessment. All risk types with threshold interest rates less than $r_2$ are effectively excluded (shown by the shaded area in Figure 1). Thus, when risk assessment is increased, as depicted in diagram (b), those investors with interest rates between $r_{2\text{min}}$ and $r_2$ will no longer find it profitable to invest. There is no a priori reason why the number of new borrowers due to risk assessment (i.e. those lying between $r_1$ and $r$) will be greater than the number of old borrowers that have been lost. However, those gained will have a lower probability of default than those lost, and so this displacement produces a less risky loan portfolio for the bank (demonstrated in the diagram by the ‘worst of the Good Risks’ being to the left of the ‘best of the Bad Risks’). Note that if $N_i$ is uniformly distributed, or monotonically increasing across $i$, the displacement results in an unambiguous utility gain for the lender since the number of borrowers displaced in higher risk subsets of $P$ will outweigh the number displaced in lower risk subsets.
Figure 1 The Favourable Selection of Risk Assessment

(a) $v^* = 1$

$\text{Good Risks}$

$\text{Bad Risks}$

(b) $v^* = 3$

worst of the Good Risks

best of the Bad Risks
**Proposition 2.1:** Increased price differentiation can have an adverse selection effect for non-uniform risk distributions

It is possible that the overall selection effect caused by risk pricing is adverse if the distribution of risks is positively skewed, causing the number of “worst of bad” risks to be proportionately greater than the number of “worst of good” risks. This is illustrated in Table 1, which offers a worked example based on a market with five risk types (probabilities of default = 0.05, 0.1, 0.2, 0.25, and 0.9 respectively). The distribution of 100 potential borrowers between the risk types is given in row three (10, 10, 10, 50 and 20 thousand respectively). Assume that there is initially a pooled interest rate such that the first three risk types are screened out (indicated by the shaded squares of rows A and B). From the respective default probabilities and numbers of applicants of risk types 4 and 5 (i.e. those not screened out), the lender can compute the number of expected defaults (12.5 and 18 respectively), leading to a total default rate of 44% on all loans.

Suppose the lender then carries out risk assessment that allows it to correctly place borrowers in one of two risk categories, and the option to charge separate interest rates. Suppose also that if the lender does this, risk types 1 and 4 will be screened out, and the remaining risk types find it profitable to take the lender’s loan offer. This leads to a total number of defaults of 21 out of 40 loans issued, a default rate of 53%, which is higher than the default rate when there was a single pooled interest rate. This numerical example is shown graphically in the first graph of Figure 2. The subsequent graphs depict the changes to the default rates as the distribution of risks
changes, the final graph illustrates how favourable selection begins to occur as the
distribution of risks flattens.

Table 1 Worked Example of Adverse Selection

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Category</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of default</td>
<td>0.05</td>
<td>0.1</td>
<td>0.2</td>
<td>0.25</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>Number of Potential Borrowers (000s)</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>50</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>Pooled r</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A. Number of Actual Borrowers (i.e. not screened out) 000s</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>50</td>
<td>20</td>
<td>70</td>
</tr>
<tr>
<td>B. Number of defaults (000s)</td>
<td>0.5</td>
<td>1</td>
<td>2</td>
<td>12.5</td>
<td>18</td>
<td>30.5</td>
</tr>
<tr>
<td>Proportion of loans that default (B/A)</td>
<td>0.44</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Separate r</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C. Number of Actual Borrowers (i.e. not screened out) 000s</td>
<td>0</td>
<td>10</td>
<td>10</td>
<td>0</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>D. Number of defaults (000s)</td>
<td>0.5</td>
<td>1</td>
<td>2</td>
<td>12.5</td>
<td>18</td>
<td>21</td>
</tr>
<tr>
<td>Proportion of loans that default (D/C)</td>
<td>0.53</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 2 Graphical Representation of Numerical Example

(a) Risk Distribution => Adverse Selection
(% defaults for pooled r = 41%)
(% defaults for separate r = 53%)

(b) Risk Distribution => Adverse Selection
(% defaults for pooled r = 44%)
(% defaults for separate r = 53%)
Risk Distribution => Adverse Selection
(% defaults for pooled $r = 44\%$
(% defaults for separate $r = 45\%$)

Risk Distribution => Fav. Selection
(% defaults for pooled $r = 51\%$
(% defaults for separate $r = 43\%$)
4.2 Costly Risk Assessment

Now assume that there is a cost schedule associated with assessing risk, $\zeta$, where:

$$v^* = v^*(\zeta); \quad v^*(\zeta) > 0; \quad \text{and} \quad v^*(\zeta = 0) = 1.$$

Banks will invest in risk assessment to the extent that the marginal gain just equals the marginal loss. Thus, factors which cause the gains to rise relative to costs, will result in a higher optimum level of risk assessment, and visa versa. The optimum level of risk assessment is denoted by $\zeta^*$.

**Proposition 3:** There exists an absolute limit for $\zeta^*$ given by $\zeta^*$ so that $0 \leq \zeta^* \leq \zeta^{-}$.

**Proof:** This follows from the assumption that there exists some level of risk assessment that results in $P$ becoming a family of singletons (that is, no more than one $p_i$ in each $P_v$), and that the bank knows when it has reached this level of risk assessment (the bank can deduce this from the fact that it knows the range of $R^e$ in each $P_v$, and so it knows that there is only one $p_i$ in $P_v$ when $R^e_{v_{\text{max}}} = R^e_{v_{\text{min}}}$ for all $v$). Beyond this level of expenditure, the bank gains nothing from additional investment in assessment.

**Proposition 4:** Only when risk assessment is sufficient to produce “near perfect” information will equilibrium credit rationing be precluded.

**Proof:** “Near perfect” information is defined as the situation where the partition of $P$ is fine enough to include only one $i$ in each partition (as is the case when $\zeta^* = \zeta^{-}$). Until the lender has achieved this level of risk assessment it will always have at least
one risk category where it has to pool different risk types and where S&W type credit rationing is possible (the proof follows from applying the same arguments outlined in the paragraphs that follow equation [6]). However, once “near perfect information” is reached, lenders can charge separate interest rates to each $i$ are therefore able to respond to excess demand for funds in any category $p_i \in P_v$ by raising the interest rate in that category, without risk of adverse selection, provided the interest rate is not raised above $r_{i\text{th}}$. If $r_v$ is raised above $r_{i\text{th}}$ then no investor in $P_v$ will apply. Thus, under “near perfect information” every risk type is treated as a separate market, each market having homogenous-risk loan applicants and an interest rate determined through the traditional interaction of demand and supply.

5 Implications of Results and Suggestions for Future Research

Niche Products

Although the possibility of adverse selection may make it sub-optimal for lenders to risk price a mainstream product, it does not preclude the emergence of pseudo-risk pricing through the development of niche products targeted at specific ranges of the risk spectrum, particularly borrowers lying at the extremes. The effect of introducing successive rounds of more refined risk categorisation is demonstrated in Figure 3 which introduces further risk categories to the diagram used in Figure 1. It can be seen that the very best risks are always screened out (depicted by shaded regions) except for the limiting case explored above where lender’s risk classification is so refined that there is only one risk type in each category and only one interest rate charged to each borrower. Conversely, the very worst risks are always screened in.
Thus, the fate of those at the extremes of the risk spectrum are for the most part unaffected by changes in risk assessment and risk pricing. As a result, there exists no ambiguity regarding the selection effect of risk pricing for these two extreme categories of borrower. Those who are always screened out because of their low risk and hence low return, comprise a niche market, ripe for ‘cherry picking’. Those at the other extreme are always screened in by interest rates, and so are also a clearly identifiable niche group whose demand for mortgage finance is likely to remain unrealised by mainstream products, opening the way for custom products to be developed specifically for this group. This is to some extent borne out by the recent entrance of new lenders into the UK mortgage market offering either very low interest rates to low-risk groups (dubbed ‘cherry-picking’ by the financial press-- see Goldsmith 1994, Pandya 1997, Scott 1995, Hunter 1995, and Berwick 1999), or high interest mortgages to particularly bad risks (‘impaired credit market’ – see Berwick

Uncertainty over the Distribution of Risks

It has been shown in the previous section that, for non-uniform risk distributions, risk pricing can cause adverse selection, making the financial case for risk pricing more ambiguous. However, even where there is a uniform distribution of risks, lenders may remain reluctant to price risks if they are uncertain of the true shape of the distribution. It is possible, for example, that the numbers of potential borrowers in each of the risk categories could vary considerably over time. So although the lender may have some working estimate that points to a uniform distribution of risks, an added layer of uncertainty in the lending decision may deter lenders from actually implementing risk pricing. A similar outcome may arise if the lender is unable to clearly distinguish risk categories. If risk assessment procedures can only place a borrower in the correct risk category with a probability less than unity, then cet par, the narrower the risk category, the lower the accuracy. It may be that in some markets, lenders can allocate risks more effectively than in others because of well established and easy to measure relationships between observable client characteristics and anticipated probability of default. If this is true of mortgage markets, lenders may not apply risk pricing because they are not confident of their ability to allocate risk appropriately. Or it may be that the story told in this paper holds true: that they can categorise risks but know that the distribution of risks is positively skewed and that adverse selection is the likely outcome. Either way, explicit knowledge of the adverse selection effect is not needed to produce an
aversion to risk pricing: lenders may simply know from experience that its introduction in certain circumstances does not optimise profits. Other factors, such as anticipated negative publicity, only compound their reluctance.

Further Unexplored Avenues

Possibilities that have not been explored in the above model but which warrant further investigation include:

(1) Lenders varying collateral requirements in conjunction with interest rates to produce an incentive compatible lending strategy. This has been explored by Bester (1987) in a pooled interest rate model with no risk assessment. However, the implications have not been modelled for lenders who have the option to assess risk directly (such as through credit scoring) and charge differentiated interest rates. One avenue for future research, therefore, would be to develop a model of lending that fully endogenises not only interest rates but also the collateral requirement and also the classification of risks/differential pricing.

(2) An additional complicating factor is the existence of credit insurance. This exists in various forms in different markets. In the mortgage market, for example, there are Mortgage Indemnity Guarantees (which insure the lender against losses made in the event of default), and Mortgage Payment Protection Insurance (which insures the borrower against repayment difficulties due to ill-health or unemployment). The effect of these products on credit rationing
and risk assessment have yet to be explored in the literature and offer another avenue of future research.

(3) The model of risk assessment and risk pricing developed above was based on discrete classifications of risk and interest rates. In certain contexts, however, it may be more appropriate to model risk assessment as a continuous process—resulting in specific estimates of default probabilities for each borrower, each estimate having an associated standard error. This raises the question of whether increased risk assessment is best thought of as an activity that reduces the standard errors on risk estimates, and whether this kind of heteroskedasticity in risk assessment has particular theoretical implications for optimal lending behaviour. Again these questions have not, to the author’s knowledge, been explored in any depth in the existing theoretical real estate literature.

5 Conclusion

In this paper I have considered the conditions under which risk pricing may not be advantageous to lenders, the awareness of which may partly explain the absence of fully risk-priced mortgages (and other financial products). In so doing, the paper has also ventured to bridge the gap between the risk assessment literature and the credit rationing literature by considering the implications of classificatory risk assessment for the S&W (Stiglitz and Weiss 1981) model. The paper began with a discussion of the emergence of credit scoring and risk-pricing and an overview of the credit rationing and risk assessment literatures. A discrete version of the S&W model was
then developed which demonstrated that raising the rate of interest causes adverse selection when there is no risk assessment, providing a rationale for equilibrium credit rationing. Risk assessment was then introduced into the model and it was shown that risk assessment, and its corollary, differentiated interest rates, will always increase the return on loans to a borrower of particular risk type. However, it was also shown how pricing of loans based on risk category can have a selection effect, producing favourable selection if the number of borrowers is uniformly distributed across risk categories and producing adverse selection if the distribution has positive skew, for example.

The rationale for favourable selection was that the borrowers “screened out” by the introduction of risk-pricing would on average have higher default probabilities than those “screened in” because the worst of the good are better than the best of the bad. The rationale for adverse selection is that if the number of “worst of good” risks is significantly greater than the number of “best of bad” risks, and it “worst of good risks” are screened out by the risk pricing, the lender may find itself receiving loan applications only from the extremes of the risk spectrum: i.e. the best of the good and the worst of the bad. If the latter group outnumbers the former (for instance, where the distribution has positive skew), then adverse selection can occur. This provides an additional explanation for lenders’ reluctance to introduce risk-pricing, and may prove to be the deciding factor in markets such as the UK mortgage market which have so far resisted the introduction of fully differentiated products. The paper also demonstrated that there is an absolute limit for optimal risk expenditure, and that S&W will be possible until this limit is reached. Thus, even when risk pricing is implemented by lenders, equilibrium credit rationing is not precluded, except in the
extreme case of near perfect information where the lender’s risk assessment is so refined as to allow it to allocate each borrower type to a unique category. The paper also discussed how the model may indirectly provide a rationale for the marketing of niche products targeted at the extremes of the risk spectrum.
References:


Notes:

1 Credit scoring techniques were initially used in the US in the 1940s to aid decisions as to whether an applicant was creditworthy, but did not become popular until the late 1960s (Andrew, 1997). The UK credit industry started to use credit scoring in the 1970s and now ‘almost all decisions to open personal bank accounts, issue a bank or credit card, or lend money to individuals, use credit-scoring as part of the process’ (ibid).

2 The early attempts at solving the credit rationing puzzle tried to find a solution within a full-information framework and tended to examine what Clemenz (1986) described as Type I rationing; that is, where ‘some or all loan applicants get a smaller loan than they desire at the quoted loan rate of interest’. More recent models have tended to consider what Clemenz classifies as Type II rationing: ‘some loan applicants are denied a loan even though for the bank they are indistinguishable from accepted applicants’ (p. 18). It is the possibility of credit rationing in the presence of perceived homogeneity of applicants that has proved to be of most interest, hence the shift of emphasis towards it. Another characteristic of the early attempts to explain credit rationing was their assumption that borrowers had different wealth endowments, and hence different capacities to offer collateral. Studies which employed this core assumption include Hodgman (1960), Freimer and Gordon (1965), Jaffee (1971), Jaffee and Modigliani (1969, 1976), Smith (1972), and Azzi and Cox (1976). These studies attempted to show, for example, that the probability of default was greater for larger loans and that this may lead the bank to restrict the size of loans to certain borrowers. A general weakness of these studies, however, was a failure to explicitly model the demand side. When consideration of demand was fully taken into account, it became impossible to demonstrate the optimality (and hence potential for equilibrium) of rationing. Adjusting price or offering separate prices to the different classes of borrowers, always proved more profitable to the lender than restricting quantity.

3 Such a relationship between risk and return is typified by the decision of the borrower whether to use K to purchase a fairly small property (e.g. sufficient to accommodate one tenant) in an already established (gentrified) area where the rental stream (either cash or imputed) is constant but at a moderate level (i.e. low risk, low return); or to purchase a larger property (e.g. sufficient to house several tenants) in an area that is as yet relatively low prices but perceived by the borrower to be ‘on the way up’ and so has the potential to earn much higher total rental income (high risk, high return).
This is to preclude the possibility that \( i+1 \) has \( R_{i+1}^i \approx R_i^i \), and \( p_{i+1} \ll p_i \) which suggests the possibility that expected profits in equation [1] may actually be less for \( i+1 \) than for \( i \). Stated in the positive, I assume that for a given rate of interest, \( p_i \) and \( R_i^i \) are related in such a way that expected profits are higher for higher risks. This is less restrictive than assuming a mean preserving spread (as in S&W op cit) but is sufficient to reproduce the S&W result.

\[ R_i^i > (1+r)K \Rightarrow r < \frac{R_i^i}{K} - 1 \]