4 F-Tests

Reading:
Kennedy (1998) "A Guide to Econometrics", Chapters 4 and 6
Maddala, G.S. (1992) "Introduction to Econometrics" p. 170-177
Moore and McCabe, chapter 12 is also worth looking at since it gives a good explanation of the more general use of F-tests (such as comparing means across several populations).
Field 2001 section 4.1.3

Aim:
The aim of this section is to consider the use of F-tests in regression analysis.

Objectives:
By the end of this chapter, students should be aware of the basic formula for F-tests run in the context of regression analysis; be able to test for the existence of a relationship; test for structural breaks and test for a set of linear restrictions.

Plan:

4.1 Introduction

It just so happens that the ratio of two residual sums of squares has an F distribution. A number of tests have been developed that make use of this principle. Since they all are based on the same basic formula (one that compares an "*unrestricted model*" with a "*restricted model*"), it makes sense to gather the most common F-tests into one place. So the purpose of this chapter is to introduce F-tests and their application to testing for the existence of a relationship, the occurrence of structural breaks, and the effect of linear restrictions. It is particularly recommended that students familiarise themselves with the procedures for testing for structural breaks since these are most useful as a diagnostic tool.

4.2 Testing a set of linear Restrictions - The General Procedure

Suppose we want to test whether there are any country specific effects in the relationship between inflation and the money supply. We can do this by testing the following null hypothesis:

H_0: $g_1 = g_2 = g_3 = \ldots = g_{42} = 0$

where g_1, g_2,...., g_{42} are the slope coefficients on country dummies in the following regression:

$INFL = a + b\ MS + g_1\ COUNTRY1 + \ldots + g_{42}\ COUNTRY42$ [1]

We can conceive of this as being equivalent to comparing two regressions, one *Restricted* and one *Unrestricted*. The *Unestricted* regression is equation [1] above with the country dummies included. The *Restricted* regression, on the other hand, is:

$INFL = a + b\ MS$ [2]

To compare these two regressions (equation [1] and equation [2]) we use an *F* test. The **general formula** for F is given by:

$$F^{df_{numerator}}_{df_{denominato}} = F^{r}_{df_U} = \frac{(RSS_R - RSS_U)/r}{RSS_U / df_U}$$

Where:

RSS_U = unrestricted residual sum of squares
        = RSS under H_1

RSS_R = restricted residual sum of squares
        = RSS under H_0

r = number of restrictions
        = difference in the number of parameters between restricted and unrestricted equations

df_u = df from unrestricted regression = $n - k$ where k is all coefficients including the intercept.

4.3 Using the F-test:

If the null hypothesis is true (i.e. restrictions are satisfied) then we would expect the restricted and unrestricted regressions to give similar results. In other words, RSS_R and RSS_U will be similar, so we accept H_0 when the test statistic gives a small value for *F*. In contrast, if one of the restrictions does not hold, then the restricted regression will have had an invalid restriction imposed upon it and will be mispecified. This results in more residual variation, and a higher restricted residual sum of squares (**NB** RSS_R will **always** be greater than RSS_U since imposing a restriction on an equation can never reduce the residual sum of squares). Consequently, we reject H_0 when the test statistic gives a large value.

4.4 Test Procedure:

There are four basic steps to executing an F-test:
(i) Compute RSS_U

Run the unrestricted form of the regression in SPSS and take a note of the residual sum of squares $= RSS_U$

(ii) Compute RSS_R

Run the restricted form of the regression in SPSS and take a note of the residual sum of squares $= RSS_R$. If the restrictions are not homogenous (i.e. if they imply that the restricted and unrestricted regressions have different dependent variables) then you need to:
  ❑ substitute the restrictions into the equation;
  ❑ rearrange the equation so that each parameter appears only once;
  ❑ create new variables where necessary and estimate by OLS;
  ❑ take a note of the residual sum of squares from this regression $= RSS_R$;

(iii) Calculate r and df_U

r is the number of restrictions and is equal to the difference in the number of parameters between the restricted regression and the unrestricted regression. df_u is the degrees of freedom from unrestricted regression $= n - k$ where k is all coefficients including the intercept. The df_u is listed in the ANOVA table (for the unrestricted regression) as the degrees of freedom of the residual. For example, in the following ANOVA table, the degrees of freedom is 511. Note also that $n = 516 =$ degrees of freedom of the Total Sum of Squares + 1 and that k (as we have defined it, i.e. the total number of coefficients including the intercept) $= 5 =$ degrees of freedom of the Regression Sum of Squares + 1.

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	296.772	4	74.193	20.652	.000[a]
	Residual	1835.811	511	3.593		
	Total	2132.583	515			

a. Predictors: (Constant), CNTRY_3, CNTRY_2, CNTRY_1, money supply

b. Dependent Variable: inflation

(iv) Substitute RSS_U, RSS_R, r and df_U

Once you have computed RSS_U, RSS_R, r and df_U, enter them in the equation for F and find the associated significance level.

NB you can find the sig value of any F value that you have calculated by using the following syntax in Excel:
$=FDIST$(F value that you've calculated, $df_{numerator}$, $df_{denominator}$)
Alternatively, you can use the Excel Template which will calculate the test F value and sig level for you from SPSS ANOVA tables (see below).

4.5 Linear restrictions where algebraic manipulation is involved (advanced)

F-tests can not only be used to test whether a group of variables in the model are jointly significant. They can also be used to test more sophisticated restrictions on the values of particular coefficients. You might, for example, wish to test whether two coefficients are equal, or whether a number of coefficients sum to one, or whether two different regressions have similar coefficients. To do these tests you need to do some algebraic manipulation of your original regression equations, as the following examples demonstrate:

Example 1

In a regression of x_2 and x_3 on y, test the hypothesis that the coefficients sum to one.

The *Unrestricted* regression is given by:

$$y = b_1 + b_2 x_2 + b_3 x_3 + u$$

The null hypothesis is H_0: $b_2 + b_3 = 1$;

Now we need to substitute the null hypothesis into the equation to derive the restricted regression. If H_0 is true, then: $b_3 = 1 - b_2$ and:

$$y = b_1 + b_2 x_2 + (1 - b_2) x_3 + u$$
$$= b_1 + b_2 x_2 + x_3 - b_2 x_3 + u$$
$$= b_1 + b_2 (x_2 - x_3) + x_3 + u$$
$$y - x_3 = b_1 + b_2 (x_2 - x_3) + u$$

Restricted regression can now be written as:

$$z = b_1 + b_2 (v) + u \qquad \text{where } z = y - x_3; \text{ and } v = x_2 - x_3$$

Example 2

For the same regression as the one considered in Example 1, test the hypothesis that $b_2 = b_3$

First note that this is a homogenous restriction because, as we shall see in a moment, the restricted and unrestricted regressions have the same dependent variable.

Unrestricted regression:

$$y = b_1 + b_2 x_2 + b_3 x_3 + u$$

H_0: $b_2 = b_3$;

If H_0 is true, then:

$$y = b_1 + b_2 x_2 + b_2 x_3 + u$$
$$= b_1 + b_2 (x_2 + x_3) + u$$

Restricted regression is simply:

$$y = b_1 + b_2 (w) + u \qquad \text{where } w = x_2 + x_3;$$

Example 3

In a regression of : MS_GDP and MP_GDP on Inflation, test the hypothesis that the coefficients on MS_GDP equals that of MP_GDP plus one.

In this case, we will find that Restricted and Unrestricted regressions have the different dependent variables and so the restrictions are not homogenous:

Unrestricted regression:
$$Infl = b_1 + b_2MS\_GDP + b_3MP\_GDP + u$$
$H_o: b_3 = b_2 + 1$
If H_0 is true, then:
$$Infl = b_1 + b_2MS\_GDP + (b_2+1)MP\_GDP + u$$
$$= b_1 + b_2MS\_GDP + b_2\,MP\_GDP + 1{\times}MP\_GDP + u$$
$$= b_1 + b_2(MS\_GDP + MP\_GDP) + MP\_GDP + u$$
$$Infl - MP\_GDP = b_1 + b_2(MS\_GDP + MP\_GDP) + u$$

Restricted regression:
$$z = b_1 + b_2(v) + u$$
where,
$$z = Infl - MP\_GDP\ ;$$
$$v = MS\_GDP + MP\_GDP$$

4.6 Using the Excel Template

You should familiarize yourself with calculating F-tests manually, but to speed calculation in routine tests and to improve precision I have developed an Excel spreadsheet which can be used to run F tests. Open up the **Lab 4 F-tests** spreadsheet, and open up the first worksheet (tab label: **General Test for Linear Restrcs**). The general F-test procedure using this spreadsheet is as follows:

(i) Run the *Unrestricted* regression and copy the ANOVA table from this regression. Place the cursor in cell D18 of the spreadsheet (the green shaded square with ANOVA written in it) and paste. This will displace any existing entry with your new ANOVA *Unrestricted* regression results.

(ii) Now run the *Restricted* regression and copy the ANOVA table from this regression. Place the cursor in cell D7 of the spreadsheet (the green shaded square with ANOVA written in it) and paste. This will again displace any existing entry with your new ANOVA *Restricted* regression results.

(iii) Excel should automatically calculate the r, df_U, F and significance level (see cells L28 to L40) but you should check the answer manually the first few times to ensure that you are using the spreadsheet correctly and that you understand the procedure. Remember that the null hypothesis is that there is no difference between the restricted and unrestricted regressions. Therefore, if F is large and the significance level small, you can reject the null with only a small probability of a Type I Error (i.e. that you have rejected H_0 incorrectly).

1. Create dummies for the first **ten** countries.

We created dummies before by simply creating a new variable with all values equal to 0, then using the If statement to set the values equal to one if the property is satisfied:

e.g. COMPUTE COUNTRY1 = 0.
    EXECUTE.
    IF(COUNTRY = 1) COUNTRY1 = 1.
    EXECUTE.

The disadvantage with this approach is that it does not take account of missing values. If a missing value exists in the variable we are trying to recode, then the dummy variable will have the value zero. However, strictly speaking, this is incorrect, since it suggests that we know that the observation does not have the attribute in question (in this case, the country is not Argentina), when in reality we do not actually know what the country is. An additional problem is that the above procedure did not encourage us to include a variable label. This means that if we do not go back and label the newly created dummy variable we can end up not knowing what it means or how it was calculated (remember that a variable *name* can only have eight characters and it can sometimes be difficult to describe a variable with so few letters).

A better approach would be to use the RECODE syntax or to use the Transform/Recode/Into Different Variables… window. The advantage of using the syntax rather than the windows procedure is that if you are creating lots of dummies you can copy and paste the syntax and only alter the parts of the syntax that need to change. It also allows you to keep a record of your work.

The recode syntax for creating the first three country dummies is as follows (the shaded syntax indicates the what needs to be amended when you create a new country dummy variable) :

```
RECODE
 country
 (1=1) (MISSING=SYSMIS) (ELSE=0) INTO Country1 .
VARIABLE LABELS Country1 'Dummy for Argentina'.
EXECUTE .

RECODE
 country
 (2=1) (MISSING=SYSMIS) (ELSE=0) INTO Country2 .
VARIABLE LABELS Country2 'Dummy for Bolivia'.
EXECUTE .
```

```
RECODE
  country
  (3=1)  (MISSING=SYSMIS)  (ELSE=0)  INTO  Country3 .
VARIABLE LABELS Country3 'Dummy for Brazil'.
EXECUTE .
```

You can copy and paste these into the SPSS syntax window and run as normal.

2. Test whether there are country effects for Argentina, Bolivia and Brazil in a regression of money supply on inflation:
$$INFL = a + b\ MS + g_1\ COUNTRY1 + g_2\ COUNTRY2 + g_3\ COUNTRY3$$

3. (Optional) In a regression explaining imports per capita (mp_pc), test the hypothesis that the coefficients on GDP per capita (gdp_pc) and exports per capita (xp_pc) sum to one. That is, for:
$$mp\_pc = b_1 + b_2\ gdp\_pc + b_3\ xp\_pc$$
   test the null hypothesis that $b_2 + b_3 = 1$

   Hint: first you need to create the new variables:

   ```
   compute xp_pc = xp/pop.
           VARIABLE LABELS xp_pc 'exports per capita'.
           execute.

           compute gdp_pc = gdp/pop.
           VARIABLE LABELS gdp_pc 'GDP per capita'.
           execute.

           compute mp_pc = mp/pop.
           VARIABLE LABELS mp_pc 'Imports per capita'.
                   execute.
   ```

   Then run the unrestricted regression (mp_pc = $b_1 + b_2$ gdp_pc + b_3 xp_pc).

   Then you need to derive the restricted regression. Before you know what regression to run, you need to work through the algebraic implications of the null hypothesis:
   ❑ substitute the restrictions into the equation (i.e. substitute $b_2 + b_3 = 1$ into the original unrestricted equation: mp_pc = $b_1 + b_2$ gdp_pc + b_3 xp_pc).
   ❑ rearrange the equation so that each parameter appears only once
   ❑ create new variables where necessary and estimate by OLS
   Once you've done this, and you know what the restricted regression looks like, run the restricted regression.

   Then paste the ANOVA tables from the restricted and unrestricted regressions into the Excel template. If the significance level is small, you can reject the null that $b_2 + b_3 = 1$. You should check your results by calculating the F-ratio manually.

4. (Optional) Test whether, in the previous question, the two slope coefficients are equal.

4.7 Testing a set of linear Restrictions when the Restrictions are Homogenous

When the linear restrictions we want to test are homogenous:

$\quad\quad$ e.g. H_0: $b_2 = b_3 = 0$

$\quad\quad$ e.g. H_0: $b_2 = b_3$

we will find that we will not need to transform the dependent variable of the restricted equation in order to specify the restricted equation.

For restrictions of this type, where the dependent variable is the same in the restricted and unrestricted regressions, we can re-write our F-ratio test statistic in terms of R^2s:

$$F^{df_{\text{numerator}}}_{df_{\text{denominator}}} = F^{r}_{df_U} = \frac{(R_U^2 - R_R^2)/r}{(1 - R_U^2)/df_U}$$

where:

$R^2_U = R^2$ from the unrestricted regression,

$R^2_R = R^2$ from the restricted regression

r = number of restrictions = difference in the number of parameters between restricted and unrestricted equations.

df_u $\quad$ = degrees of freedom from unrestricted regression = $n - k$ where k is the total number of coefficients in the unrestricted regression including the intercept.

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5. (Optional) Look again at the restrictions suggested in the previous questions and consider whether they are homogenous or not. If they are homogenous, try running the test again, but this time use the simplified F-ratio test statistic.

4.8 Testing for the existence of a relationship

If we are testing a set of linear restrictions where the restrictions are that all slope coefficients are zero ($b_j = 0$ for all j), in other words we are testing for the existence of a relationship, then we can simplify the F-ratio test statistic further. Suppose the *Unrestricted regression* is as follows:

$\quad\quad y = b_1 + b_2x_2 + b_3x_3 + u$

$\quad\quad H_0$: $b_2 = b_3 = 0$;

If H_0 is true, then $y = b_1$. In this case, the *Restricted regression* does no explaining at all and so $R_R^2 = 0$. As a result, the homogenous restriction F-ratio test statistic reduces to:

$$F^{df_{\text{numerator}}}_{df_{\text{denominator}}} = F^{r}_{df_U} = \frac{(R_U^2 - 0)/r}{(1 - R_U^2)/df_U}$$

$$= \frac{(R_U^2)/r}{(1 - R_U^2)/df_U}$$

where,

$\quad df_u$ $\quad$ = $\quad$ degrees of freedom from unrestricted regression = $n - k$ where k is the total number of coefficients in the unrestricted regression including the intercept.

$\quad R^2_U$ $\quad\quad$ = $\quad$ R^2 from the unrestricted regression,

$\quad r$ $\quad$ = $\quad$ number of restrictions = $k - 1$.

6. This is the F-test we came across in Session 2, and is the one automatically calculated in the SPSS ANOVA table. Try calculating the test manually and compare it with the ANOVA table output.

4.9 Testing for Structural Breaks

The *F*-test also comes into play when we want to test whether the estimated coefficients change significantly if we split the sample in two at a given point. These tests are sometimes called "Chow Tests" after one of its early proponents. There are actually two versions of the test: *Chow's First Test* and *Chow's Second Test*.

Chow's First Test
Use where $n_2 > k$

(1) Run the regression on the first set of data ($i = 1, 2, 3, \ldots n_1$) and let its RSS be labeled RSS_{n1}
(2) Run the regression on the second set of data ($i = n_{1+1}, n_{1+2}, \ldots$, end of data) and let its RSS be labeled RSS_{n2}
(3) Run the regression on the two sets of data combined ($i = 1, \ldots$, end of data) and let its RSS be RSS_{n1+n2}
(4) Compute RSS_U, RSS_R, r and df_U where:
   $RSS_U = RSS_{n1} + RSS_{n2}$
   $RSS_R = RSS_{n1+n2}$
   $r \quad = k = \quad$ total number of coefficients including the constant
   $df_U \quad = \quad n_1 + n_2 - 2k$
(5) Use RSS_U, RSS_R, r and df_U to calculate F using the general formula for F and find the sig. Level:

4.9.1.1   Chow's Second Test
Use where $n_2 < k$ (I.e. when you have insufficient observations on the second sub-sample to run Chow's 1st test)
(1) Run the regression on the first set of data (i = 1, 2, 3, … n1) & let its RSS be RSSn1
(2) Run the regression on the two sets of data combined (i = 1, …, end of data) & let its RSS be RSSn1 + n2
(3) Compute RSSU, RSSR, r and dfU:
   RSSU = RSSn1
   RSSR = RSSn1 + n2
   r   = n2
   dfU   = n1 – k
   (4) Use RSS_U, RSS_R, r and df_U to calculate F using   the general formula for F and find the significance:

7. In the imports per capita regression you estimated above, use Chow's First Test to test the hypothesis that slope coefficients changed (i.e. there was a structural break) before and after 1986.

8. Test the same hypothesis using Chow's Second Test. Which test is more reliable do you think in this instance?

9. In the same regression, test the hypothesis that the slope coefficients are different for smaller countries (pop < 15).